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Normal Filters and Fodor's theorem

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Abstract

We study filters on an uncountable cardinal κ which are closed by diagonal intersection. A generalisation of Fodors pressing down lemma is at the base of subsequent results. Filters are extended to normally closed filters by iterating diagonal intersections. Conditions for a κ-complete filter on κ to be estensible to a non-trivial normally closed filter are given.

1 Introduction

This paper is the report of the internship I spent at University of Calgary from june to august 1995. The department of Mathematics and Statistics employs mathematicians and graduate students working in various domains as logics and graph theory, algebra, topology, applied mathematics and approximation theory, statistics.

The main subject I was studying is set theory. The axioms of Zermelo-Fraenkel enable a definition of natural numbers and more generally ordinals. Ordinal arithmetic extends natural number arithmetic, and the principle of transfinite recursion, the recursion over the natural numbers. I had the opportunity to see some model theory, the relevance of certain axioms as for example the axiom of regularity. My job was to read lecture notes on set theory given by Eric Milner at University of Calgary and to verify the proofs.

Cardinal numbers, cofinality, cardinal arithmetic and inequalities are fundamental notions on which relies the theory of clubs and stationary sets. These well known results can be found in set theory books (as for example Jech, Set theory) and will not be repeated in this paper unless directly used.

The second half of my training consisted in exploring the results concerning clubs and stationary sets which are generally true for normal filters. These results are detailed in the following sections.

The main benefit of the internship was the acquisition of set theory, extending the lectures I assisted to during the last two years about logics and discrete mathematics. These notions are fundamental for many branches of discrete mathematics or theoretical computer science. Working on the particular topic of normal filters gave me the opportunity to gain some specialized knowledge in filter and set theory and to have an experience in research work.

2 Filters and ideals

Definition 1 *Let κ be an infinite cardinal number. We will call a* filter *on κ a subset $\mathcal{F}$ of $\mathcal{P}(\kappa)$ satisfying:*

1. $\forall A, B \in \mathcal{F} \;\; A \cap B \in \mathcal{F}$
2. $\forall A \in \mathcal{F},\ B \in \mathcal{P}(\kappa) \;\; A \subseteq B \;\Rightarrow\; B \in \mathcal{F}$

If $\mathcal{A} \subseteq \mathcal{P}(\kappa)$, $\mathcal{A}^*$ will denote the set $\{\kappa \backslash S \mid S \in \mathcal{A}\}$. $\mathcal{A}$ and $\mathcal{A}^*$ are said to be dual. *A subset $\mathcal{I}$ of $\mathcal{P}(\kappa)$ will be called an* ideal *iff $\mathcal{I}^*$ is a filter on κ. A filter $\mathcal{F}$ will be called* proper *if $\emptyset \notin \mathcal{F}$ or - which is equivalent - if $\mathcal{F} \neq \mathcal{P}(\kappa)$.*

If $\mathcal{A}$ is a subset of $\mathcal{P}(\kappa)$, we write $[\mathcal{A}]$ for the set

$$\{S \in \mathcal{P}(\kappa) \mid \exists n \exists A_1, A_2, \ldots, A_n \in \mathcal{A}, \bigcap_k A_k \subseteq S\}$$

$[\mathcal{A}]$ is called the filter generated by *$\mathcal{A}$.*

A filter generated by a single element is called *principal.* An important filter is the *Frechet-filter* $\mathcal{F}R$:

$$\mathcal{F}R := \{S \subseteq \kappa \mid |\kappa \backslash S| < \omega\}$$

As points contained in every element of the filter present less interest for our purpose, we will suppose from now on that all filters include $\mathcal{F}R$.

Definition 2 *Let $\mathcal{F}$ be a filter and λ a cardinal. $\mathcal{F}$ is said to be λ-*complete *iff for each cardinal $\alpha < \lambda$ any intersection of α elements of $\mathcal{F}$ belongs to $\mathcal{F}$. Similarly an ideal $\mathcal{I}$ is λ-complete iff the dual filter $\mathcal{I}^*$ is λ-complete.*

3 Normal filters

We introduce in this section normal filters, a notion which has first been used by P. Scott. It generalizes some properties of the club filter which we will detail in section 5.

We will suppose from now on that κ is a limit cardinal such that $cf(\kappa) > \omega$. Unless otherwise stated, κ doesn't have to be regular.

Definition 3 *Let $\mathcal{A}$ be a subset of $\mathcal{P}(\kappa)$ and $(X_\xi)_{\xi \in \kappa}$ a family of sets in $\mathcal{A}$. We define the* diagonal intersection *of $(X_\xi)_\xi \in \kappa$, noted ΔX_ξ, by:*

$$\Delta X_\xi := \{\alpha \in \kappa \mid \alpha \in \bigcap_{\xi < \alpha} X_\xi\}$$

$\Delta\mathcal{A}$ will denote the set of all diagonal intersections of families of elements of $\mathcal{A}$. We say that the filter $\mathcal{F}$ is normal *if $\mathcal{F}$ is closed under diagonal intersections i.e. if $\Delta\mathcal{F} = \mathcal{F}$.*

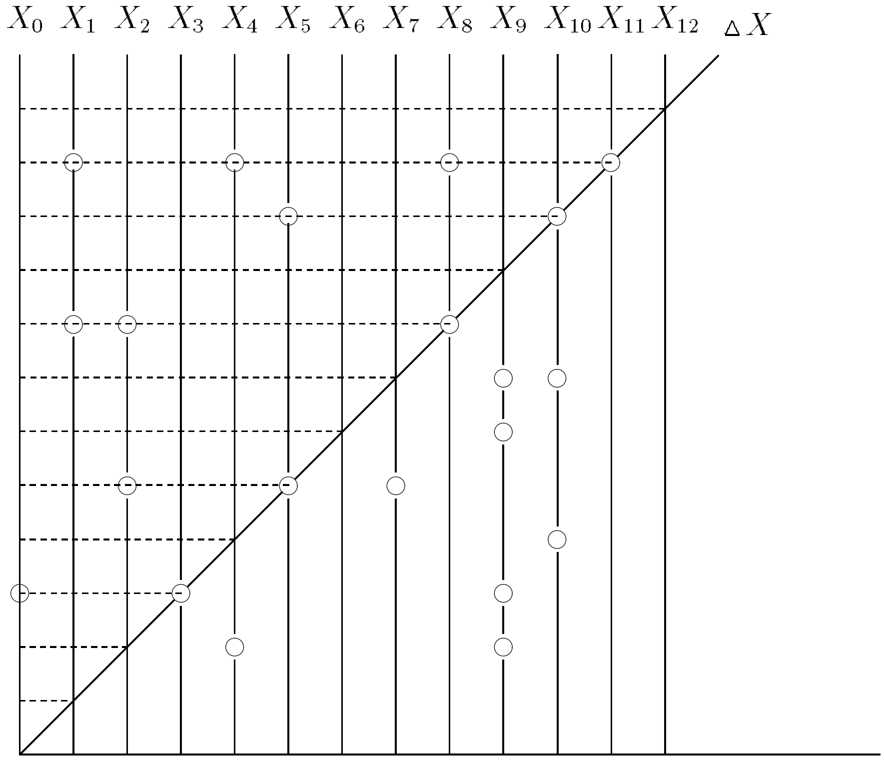

Figure 1: Diagonal intersection

One can ask for the links between normality and λ-completeness. The initial idea of normality is to provide a strengthening of κ-completeness. In the literature the definition of normal very often includes already κ-completenessness. κ-completeness does not imply normality as can easily be seen on the filter $([\kappa]^{<\kappa})^*$ of all subsets of κ whose complement has cardinality $< \kappa$. We prove here that normality implies ω_1-completeness:

Proposition 1 *If $\mathcal{F}$ is a normal filter on the limit cardinal κ then $\mathcal{F}$ is ω_1-complete.*

Proof:
Let $(X_n)_{n\in\omega}$ be a family of members of $\mathcal{F}$. We define for $n \in \omega$:

$$X'_n := \begin{cases} X_n \cap X_p & \text{with } p \text{ minimal s.t. } n{+}1 \notin X_p, \text{ if there is such a } p \\ X_n & \text{otherwise} \end{cases}$$

and for $\xi \geq \omega$, $X_\xi := \kappa$. We have then $\bigcap_{\xi \in \omega} X_\xi = \Delta X'_\xi \in \mathcal{F}$. *q.e.d.*

An attempt to generalize this proof fails. The idea would be to define $h : \kappa \to \lambda$ for a family $(X_\xi)_{\xi \in \lambda}$, such that $\alpha \notin \bigcap_{\xi \in \lambda} X_\xi \;\; \to \;\; \alpha \notin X_\alpha$. For any regressive function $f : \kappa \to \kappa$, the diagonal intersection of the family $X'_\xi := \bigcap_{\alpha \in f^{-1}(g(\xi))} X_\alpha$ is $\bigcap_{\xi \in \lambda} X_\xi$. Unfortunately we can not be sure that each X'_ξ belongs to $\mathcal{F}$.

4 "Fodor's" theorem

Fodor's theorem [Fod] generalizes a result due to Alexandroff and Urysohn [AU]. In this section we will state a further generalisation of it. Solovay, Reinhardt, and Kanamori [SRK] have proved it for the case of ultrafilters on $(\lambda)^\kappa$ and Jech [Je] for the closed unbounded filter on $\mathcal{P}_\kappa(\lambda)$.

Definition 4 *If $\mathcal{F}$ is a filter and S a subset of κ. S will be said to have* positive $\mathcal{F}$-measure, *or to be* $\mathcal{F}$-positive *if $\forall X \in \mathcal{F}$, $S \cap X \neq \emptyset$. The set of all $\mathcal{F}$-positive subsets of κ will be denoted by $\mathcal{F}^+$. If $S \in \mathcal{P}(\kappa) \backslash \mathcal{F}^+$, S has* $\mathcal{F}$-measure 0 *or is* $\mathcal{F}$-small.

One can remark that $\mathcal{F}^*$ is the set of all $\mathcal{F}$-small sets.

Definition 5 *Let S be a subset of κ and $f : S \longrightarrow \kappa$ be a function. f is* regressive *iff $\forall x \in S \backslash \{0\}$, $f(x) < x$*

Theorem 2 *Let $\mathcal{F}$ be a filter. $\mathcal{F}$ is normal iff $\forall S \in \mathcal{F}^+$, and f regressive function on S, f is constant on a $\mathcal{F}$-positive subset of S.*

Proof:

- $\Rightarrow$: Suppose $\forall \alpha \in \kappa, f^{-1}(\alpha) \in \mathcal{F}^*$. Let $X_\alpha := \kappa \backslash f^{-1}(\alpha)$. As $\mathcal{F}$ is normal $\Delta X_\alpha \in \mathcal{F}$. $S \in \mathcal{F}^+$ implies that $S \cap \Delta X_\alpha \neq \emptyset$. So let $\gamma \in S \cap \Delta X_\alpha$, we have $f(\gamma) \geq \gamma$ in contradiction with f regressive.

- $\Leftarrow$: Suppose $\mathcal{F}$ not normal, and $(X_\xi)_{\xi \in \kappa}$ a family of members of $\mathcal{F}$ such that $\Delta X_\alpha \notin \mathcal{F}$. Let $S := \kappa \backslash \Delta X_\alpha$ and $f : S \to \kappa$ defined by $f(\alpha) := \gamma$ where $\gamma < \alpha$ and $\alpha \notin X_\gamma$. f is regressive. There is an $\mathcal{F}$-positive set $S' \subseteq S$ and $\gamma \in \kappa$ such that $\forall \xi \in S'$, $f(\xi) = \gamma$. S' $\mathcal{F}$-positive implies that $\exists \beta \in S' \cap X_\gamma$, so $f(\beta) = \gamma$ but this is a contradiction with $\beta \notin X_\gamma$ by the definition of f. *q.e.d.*

5 Clubs and stationary sets

Definition 6 *A subset S of κ will be called* unbounded *if*

$$\forall y \in \kappa, \exists x \in S, \ y < x$$

S will be closed[1] *if the limit of any increasing sequence of elements of S is in S.*

If we write $\mathcal{C}$ for the set of closed unbounded subsets (*clubs*) of κ then we have:

- A finite intersection of clubs is a club.
- Consequently, as a club is necessarily non empty and infinite, $[\mathcal{C}]$ is a non-trivial filter, to which we will refer from now on as the *club filter*.
- The club filter is actually $cf(\kappa)$-complete.

These results are proved in the appendix.

Definition 7 *If $\mathcal{F}$ is the club filter, the members of $\mathcal{F}^+$ will be called* stationary *sets, and the members of $\mathcal{F}^*$* non-stationary.

What makes clubs interesting for our purpose is that the club filter is closed under diagonal intersections if κ is a regular uncountable cardinal. As a corollary of theorem 2 we have then the actual Fodor's theorem [Fod].

Corollary 3 *Fodor, 1956*
If κ is a regular uncountable cardinal and S is a stationary subset of κ, and $f : S \to \kappa$ is regressive, then there is a stationary set $S' \subseteq S$, such that f is constant on S'.

It is interesting to note that, if $\mathcal{F} = ([\kappa]^{<\kappa})^*$, then $\Delta\mathcal{F}$ is the club filter: On one hand it is clear that $\mathcal{F}$ is included in the club filter. For a proof of the opposite inclusion, consider a club C in κ. We define for each $\xi \in \kappa$,

$$X_\xi := \kappa \setminus \bigcup_{I \cap C = \emptyset} I \quad \text{where } I = [\xi + 1, \alpha], \ \alpha \text{ arbitrary}$$

[1] It is equivalent to say that S is a closed subset for the topology generated by the open intervals of κ

Each X_ξ belongs to $\mathcal{F}$ and C is the diagonal intersection of the X_ξ.

A natural question is whether $\Delta\mathcal{F}R$ equals the club filter. This is true for the case $\kappa = \omega_1$:

Proof: By [FM] for each non stationary set S in ω_1 there is a regressive function f on S assuming each of its values at most twice. If C is a club, we take $S := \omega_1 \backslash C$ and f the function defined on S according to the stated result. We take $X_\xi := \kappa \backslash f^{-1}(\{\xi\})$; C is then the diagonal intersection of the X_ξ. *q.e.d.*

The result can not be generalized to any cardinal, by the following theorem of Fodor and Máté [FM]:

Theorem 4 *If κ is a cardinal such that $cf(\kappa) > \omega$. Then there is a non-stationary set $S \subseteq \kappa$ such that, for each cardinal $\lambda < \kappa$, every regressive function on S is constant on a set of cardinality λ.*

6 Subnormality

It is easy to see that, given a family of normal filters on a cardinal κ, their intersection is still a normal filter. As $\mathcal{P}(\kappa)$ is normal, there is a minimal normal filter extending a given family $\mathcal{A}$ of subsets of $\mathcal{P}(\kappa)$, the *normal extension* of $\mathcal{A}$. A filter is *subnormal* if its normal extension is a proper filter. A different approach to the normal extension of a filter is to iterate diagonal intersections:

6.1 Iterating diagonal intersections

Given a family of sets $\mathcal{A} \subseteq \mathcal{P}(\kappa)$, we define:

$$\Delta^0\mathcal{A} := \mathcal{A}$$

$$\Delta^\lambda\mathcal{A} := \begin{cases} \Delta(\Delta^\mu\mathcal{A}) & \text{if } \lambda = \mu + 1 \\ \bigcup_{\mu<\lambda} \Delta^\mu\mathcal{A} & \text{if } \lambda \text{ is a limit ordinal} \end{cases}$$

$\Delta^{\kappa^+}\mathcal{A}$ generates the normal extension of $\mathcal{A}$, as it cannot be extended by applying the Δ-operation.

For any given $n < \omega$ and a function $g : \kappa^n \to \mathcal{A}$ we write

$$\Delta g := \{\xi \in \kappa \mid \alpha_1, \alpha_2, \ldots, \alpha_n \in \xi \ \to \ \xi \in g(\alpha_1, \alpha_2, \ldots, \alpha_n)\}$$

It is easy to see that $\Delta^n\mathcal{A} = \{\Delta g \mid g : \kappa^n \to \mathcal{A}\}$.

If $g : \kappa^{<\omega} \to \mathcal{A}$ we define

$$\Delta g := \{\xi \in \kappa \mid \alpha \in \xi^{<\omega} \ \to \ \xi \in g(\alpha)\}$$

$$\Delta^{<\omega}\mathcal{A} := \{\Delta g \mid g : \kappa^{<\omega} \to \mathcal{A}\}$$

Some preliminary remarks that can be made:

- if $\mathcal{F}$ is a κ-complete filter then $\Delta\mathcal{F}$ is the normal extension of $\mathcal{F}$: If $f : \kappa^2 \to \mathcal{F}$ we can take $g(\alpha) := \bigcap_{\gamma\leq\alpha,\beta\leq\alpha} f(\gamma,\beta)$. We have then $\Delta g \subseteq \Delta f$ as $x \in \Delta g$ and $p, q < x$ implies $x \in g(\max(p,q)) \subseteq f(p,q)$.
- By the definition, for any family $\mathcal{A}$, $\Delta^\omega\mathcal{A} \subseteq [\Delta^{<\omega}\mathcal{A}]$.

Theorem 5 *If $\mathcal{F}$ is a filter then $\Delta^{<\omega}\mathcal{F}$ is a normal filter.*

Proof:
Suppose we have a family $(\Delta f_\xi)_{\xi\in\kappa}$ of members of $\Delta^{<\omega}\mathcal{F}$ defined by the functions $f_\xi : \kappa^{<\omega} \to \mathcal{F}$. We define $f : \kappa^{<\omega} \to \mathcal{F}$ by:

$$f(\emptyset) := \kappa$$

$$f(\xi, \alpha_1, \alpha_2, \ldots, \alpha_n) := f_\xi(\alpha_1, \alpha_2, \ldots, \alpha_n)$$

We prove $\Delta f \subseteq \Delta_\xi(\Delta f_\xi)$: Suppose $x \in \Delta f$. Then $\xi < x$ implies

$$\forall \alpha \in x^{<\omega} \ x \in f(\xi, \alpha) = f_\xi(\alpha)$$

So $x \in \Delta_{\xi\in\kappa}(\Delta f_\xi)$. *q.e.d.*

If $\mathcal{F}$ is a filter, $\mathcal{F}'$ its normal extension, it is easy to see that the family

$$\{\bigcap_{n\in\omega} X_n \mid \forall n \in \omega \ X_n \in \Delta^n\mathcal{F}\}$$

is contained in $\mathcal{F}'$, as $\mathcal{F}'$ is ω_1-complete and contains $\Delta^\omega\mathcal{F}$. On the other hand this family ω_1-generates [2] $\Delta^{<\omega}\mathcal{F}$: Any element Δf of $\Delta^{<\omega}\mathcal{F}$ is the intersection of the Δf_n where $f_n : \kappa^n \to \mathcal{F}$ is the restriction of f to κ^n.

As a conclusion we have that $\Delta^{<\omega}\mathcal{F}$ is the normal extension of $\mathcal{F}$.

In recent papers they study the notion of *normal depth* of a family of sets $\mathcal{A}$ which is the least ordinal μ such that $\Delta^\mu\mathcal{A}$ is a normal filter.

[2] The filter λ-generated by a family of sets $\mathcal{A}$ is $[\mathcal{A}']$, where $\mathcal{A}'$ is the family of intersections of strictly less than λ members of $\mathcal{A}$.

6.2 Conditions for subnormality

If we assume κ-completeness, we have the following caracterisation of subnormal filters:

Theorem 6 *A κ-complete filter $\mathcal{F}$ is subnormal iff $\emptyset \notin \Delta\mathcal{F}$.*

Proof:
As a matter of fact $\mathcal{F}$ is contained in its normal extension which is proper and so doesn't contain $\emptyset$.

On the other hand we make the remark in section 6.1 that if $\mathcal{F}$ is κ-complete, then $\Delta\mathcal{F}$ is the normal extension of $\mathcal{F}$. *q.e.d.*

Corollary 7 *For a κ-complete filter $\mathcal{F}$ we have equivalence between:*

(i). *$\mathcal{F}$ is subnormal*

(ii). *For any regressive function $f : \kappa \to \kappa$ there is an $\mathcal{F}$-positive set $S \subseteq \kappa$ such that f is constant on S.*

(iii). *If $S' \in \mathcal{F}$, there is no $\mathcal{F}$-small regressive function $f : S' \to \kappa$*

Proof:
$\mathcal{F}$ subnormal is equivalent by the precedent theorem to κ is $\Delta\mathcal{F}$-positive. By theorem 2 we can state the existence of an $\Delta\mathcal{F}$-positive set $S \subseteq \kappa$ such that f is constant on S. S is $\mathcal{F}$-positive as $\mathcal{F} \subseteq \Delta\mathcal{F}$. On the other hand if there is an $\mathcal{F}$-small regressive function $f : \kappa \to \kappa$ then f is $\Delta\mathcal{F}$-small and so κ is $\Delta\mathcal{F}$-small too by theorem 2. So we have (i) $\Leftrightarrow$ (ii).

As $\kappa \in \mathcal{F}$ we have that (iii) $\Rightarrow$ (ii).

Suppose $S' \in \mathcal{F}$ and $f : S' \to \kappa$ an $\mathcal{F}$-small regressive function. We extend f to a regressive function on κ by $\forall x \in \kappa \backslash S'\ f(x) := 0$. f is then $\mathcal{F}$-small on kappa, and we get a contradiction with (ii). *q.e.d.*

7 Conclusion

Future developments could include a deeper look at the links between λ-completeness, for $\omega \leq \lambda < \kappa$, and normality. It is probable that normality does not necessarily imply κ-completeness. In this case it would be interesting to have some examples of not κ-complete normal filters.

The internship at University of Calgary has had a lot of positive elements. Ideal working conditions were guaranteed mainly by the huge library of University of Calgary, where it was possible to find nearly all needed references and articles, and by the readiness of the researchers in the mathematics department to discuss questions concerning my studies.

I thank in particular Eric Milner, who accepted to be my scientific supervisor for this period and organised my stay.

If the mathematics department is generally less busy during the summer months it is true that I could meet numerous visitors spending some time at University of Calgary, from which I will cite John Truss for his very encouraging support.

A Proofs

We give here proofs of some known facts about the club filter on a cardinal κ. We always suppose $cf(\kappa) > \omega$.

Proposition 8 *The intersection of two clubs is a club.*

Proof:

Let A, B be two clubs. Then:

(i). $A \cap B$ is closed

The intersection of two closed subsets of κ is closed.

(ii). $A \cap B$ is unbounded

Let α be in κ. As A and B are unbounded we can choose an increasing sequence $(x_i)_{i \in \omega}$ with

$$x_0 > \alpha$$

and

$$x_0, x_2, x_4, \ldots, x_{2i}, \ldots \in A$$

$$x_1, x_3, x_5, \ldots, x_{2i+1}, \ldots \in B$$

We have then

$$x_\omega := \lim_{i \in \omega} x_i \in A \cap B$$

q.e.d.

Proposition 9 *The intersection of less than $cf(\kappa)$ clubs on the cardinal κ is a club.*

Proof:

(i). The intersection of less than $cf(\kappa)$ clubs on the cardinal κ is closed, as any intersection of closed subsets of κ is closed.

(ii). The intersection of less than $cf(\kappa)$ clubs on the cardinal κ is unbounded.

We prove this result by transfinite induction on the ordinal λ.

Suppose $\lambda < cf(\kappa)$. Suppose that any intersection of μ clubs, with $\mu < \lambda$, is a club. If λ is a successor ordinal, it follows by the preceding proposition that an intersection of λ clubs is unbounded. If λ is a limit ordinal, and $(X_\xi)_{\xi<\lambda}$ a sequence of λ clubs, we define:

$$\forall \mu < \lambda,\ X'_\mu := \bigcap_{\xi \leq \mu} X_\xi$$

By induction hypothesis $\forall \mu < \lambda,\ X'_\mu$ is a club and $(X'_\mu)_{\mu<\lambda}$ is a decreasing sequence. If $\alpha \in \kappa$ we can define by transfinite induction an increasing sequence $(x_\mu)_{\mu<\lambda}$ such that

$$x_0 > \alpha$$

$$\forall \mu < \lambda,\ x_\mu \in X'_\mu$$

As $\lambda < cf(\kappa),\ x_\lambda := \lim_{\mu<\lambda} x_\mu$ belongs to κ. It is clear that

$$\forall \mu < \lambda,\ x_\lambda \in X'_\mu$$

As a consequence

$$x_\lambda \in \bigcap_{\mu<\lambda} X_\mu$$

q.e.d.

Proposition 10 *If κ is an uncountable regular cardinal, then the club-filter on κ is closed by diagonal intersections.*

Proof

Let $\mathcal{F}$ denote the club-filter on the uncountable regular cardinal κ and $(X_\xi)_{\xi\in\kappa}$ be a family of members of $\mathcal{F}$. In order to prove that $\Delta_\xi X_\xi$ belongs to $\mathcal{F}$ we show that

(i). $\Delta_\xi X_\xi$ is closed

Let $(x_\xi)_{\xi\in\lambda}$ be an increasing sequence in $\Delta_\xi X_\xi$ and $x := \lim_\xi x_\xi$. Suppose $x \in \kappa$ and x a limit ordinal. We have:

$$\forall\xi \in \lambda,\ \forall\mu < x_\xi,\ x_\xi \in X_\mu$$

$$\forall\alpha < x, \exists\nu < \lambda \text{ such that } x_\nu > \alpha$$

And so the sequence $(x_\xi)_{\nu\leq\xi<\lambda}$ is included in X_α. This implies:

$$\forall\alpha < \lambda,\ x \in X_\alpha$$

and so

$$x \in \Delta_\xi X_\xi$$

(ii). $\Delta_\xi X_\xi$ is unbounded

We define:

$$X'_\mu := \bigcap_{\xi\leq\mu} X_\xi$$

As $\mathcal{F}$ is $cf(\kappa)$-complete, $\forall\mu \in \kappa,\ X'_\mu \in \mathcal{F}$. It is clear that $\Delta_\xi X'_\xi = \Delta_\xi X_\xi$ and that $(X'_\xi)_{\xi\in\kappa}$ is a decreasing sequence. If $\alpha \in \kappa$, by transfinite induction we can define a strictly increasing sequence $(x_\xi)_{\xi\in\omega}$ with

$$x_0 = \alpha$$

and

$$\forall\xi \in \omega,\ x_{\xi+1} \in X'_\xi$$

If $x_\omega := \lim_\xi x_\xi$, it is clear that $\forall\alpha < x_\omega,\ x_\omega \in X'_\alpha$ and so $x_\omega \in \Delta_\xi X_\xi$.

q.e.d.

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