Relational Quantum Mechanics

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Abstract

In this internship report, we present Carlo Rovelli's relational interpretation of quantum mechanics, focusing on its historical and conceptual roots. A critical analysis of the Einstein-Podolsky-Rosen argument is then put forward, which suggests that the phenomenon of 'quantum non-locality' is an artifact of the orthodox interpretation, and not a physical effect. A speculative discussion of the potential import of the relational view for quantum-logic is finally proposed.

Figure 0.1: Composition X, W. Kandinski (1939)

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Introduction

"We should be careful not to attribute to the heavens
what is really in the observer."
Copernicus

This memoir is concerned with the interpretation of quantum mechanics (henceforth QM). The 'problem' encapsulated in this expression – an old one, on which many pages have been written – is the following.

QM is an utterly effective instrument for computing empirical statistics. Since its discovery, almost a century ago, its range of applicability, formerly atomic physics, has kept widening beyond all limits – one now even speaks of 'quantum cosmology'! More prosaically, QM's predictions are exploited daily in engineering science, with a societal influence that needs no extra-comment. Yet, physicists do not have any answer to these naive, almost childish, question: *What is QM about? What does it mean?* It seems we simply do not know what our best physical theory is about! This mere ignorance takes the form a cruel dissensus among experts on the true 'quantum-mechanical picture of the world'. Fuchs has humorously drawn the attention of the community on this peculiar situation [18]:

> "Go to any meeting, and it is like being in a holy city in great tumult. You will find all the religions with all their priests pitted in holy war – the Bohmians, the Consistent Historians, the Transactionalists, the Spontenous Collapseans, the Einselectionists, the Contextual Objectivists, the outright Everettics, and many more beyond that. They all declare to see the light, the ultimate light. Each tells us that if we will accept their solution as our savior, then we too will see the light. But there has to be something wrong with this! If any of these priests had truly shown the light, there would simply not be the year-after-year conference. The verdict seems clear enough: If we [...] really care about quantum foundations, then it behooves us as community to ask why these meetings are happening, and find a way to put a stop to them."

How can we "put a stop" to the problem of the interpretation of QM? Carlo Rovelli may have an answer to this. Taking his cue from Einstein's conclusion of the debate on the physical meaning of the Lorentz transformations, he proposes [5]:

- "That the unease may derive from the use of a concept which is inappropriate to describe the physical world at the quantum level. I shall argue that this concept is the concept of observer-independent state of a system, or, equivalently, the concept of observer-independent values of physical quantities.
- That quantum mechanics will cease to look puzzling only when we will be able to *derive* the formalism of the theory from a set of simple physical assertions ("postulates", "principles") about the world. Therefore, we should not try to *append* a reasonable interpretation to the quantum mechanics *formalism*, but rather to *derive* the formalism from a set of experimentally motivated postulates."

In this paper of 1996, Rovelli put forward a novel view on QM, denoted Relational Quantum Mechanics (RQM). During this internship with him, I have investigated certain aspects of RQM. The presentation of this new 'interpretation', and its application to the paradigmatic 'EPR paradox', are the goals of this memoir. But before delving deeper into this matter, let us take the time to step back a little, and try to understand the physical origin of the stance called 'relationalism' – from which RQM obviously springs.

Prelude: the roots of relationalism

In the context of physics, relationalism is generally opposed to substantivalism, and mostly refers to the problem of the ontology of space (and time): is space a substance, or a relation[5]? Originating in this metaphysical question, the insights of relationalism have been at the roots of all important discoveries in fundamental physics, from

[5]For a survey of the views presented in this introduction, we refer to Rovelli's book [40]; insightful reflections on the possibility of radical relationalism are proposed in *The end of time* by Barbour [39].

Galileo's kinematics to Einstein's geometrodynamics, *including quantum mechanics.* Unfortunately, Heisenberg and Bohr's original relationalism was muddled up with a form of holism, in a mixture termed by Bohr 'complementarity'. In order to truly understand the relevance of relationalism in QM, let us have a glimpse of to the epistemological scope of this term[6].

More than a metaphysical position, which would affirm the relational nature of all quantities, relationalism can be understood as a reaction to the natural tendency of physics to create hypostases. It is indeed a common preconception of working scientists that theoretically efficient *concepts* are ontologically secured *elements of reality.* This used to be the case of the Newtonian 'absolute space'. This substantivalism, which is clearly reminiscent of an old tradition of metaphysics in Western societies (perhaps tracing back to Parmenides), is the standard (and most efficient) position during the periods of normal science, in Kuhn's sense. Nevertheless, this attitute arguably obscures the transient nature of scientific ontologies. Against this philosophical inertia, relationalism proposes a reappraisal of the operational foundations of physical *entities.* The two pillars of this therapeutic relationalism are *operationalism* and *monism.* The example of relativity theory, through the teachings of Mach and Einstein, might justify this claim.

In a seminal critique [41], Mach opposed a strict *operationalism*[7] to Newtonian mechanics. According to this doctrine, the progress of science requires the complete elimination of its remnant hypostases, such as unobservable entities or transcendent phenomena. Indeed, the goal of science is to organize *in the most economical way* the multiplicity of experimental facts, *and not* to explain them by hypostatic causes. Thus, in the Machian epistemology, a concept referring to some empirically inaccessible (but supposedly influential) reality is not only superfluous; it is prejudicial to the very mission of science.

Now, Newton's absolute space is one of those unobservable entities. When we say we measure a distance, what we really do is *compare* two material objects (one of which we call a ruler). Contrary to Newton's claim, the same remark holds for the measurement of accelerations. Just like velocities, they are defined relative to a material frame. For how could we measure them without the presence of a material apparatus? All the same, Mach argues, concepts such as 'mass' or 'force' need be given a definition connecting them to an operational procedure. In this sense, Mach's operationalism is deeply relational: physical values are meaningless, unless they can be explicitely related to their measuring apparata.

What should we retain from Mach's analysis? The application of Occam's razor to physical entities or qualities, in a pragmatic reconsideration of experimental procedures – a process which we could call "Mach's razor". The consequence of the effect of Mach's razor on the foundations of mechanics is the programmatic relativization of all kinematical quantities. Mach himself was not able to achieve this program[8], but certainly opened the door to the revolution of relativity theory.

The second pillar of relationalism is, to our understanding, *monism.* According to this ontological precept, *there is no distinction a priori between physical entities.* Since Copernicus' (re)discovery of the Earth's motion, this monistic principle has operated as a most efficient cure against anthropocentrism. Just as it guided Galileo to the acknowledgment of the relativity principle, it was essential to Einstein's special relativity: following this principle, neither the Earth nor the Ether can bear exceptional properties allowing to define such a thing as 'absolute motion'. A few years after 1905, Einstein extended the range of the principle of equivalence of physical systems to all reference frames (either inertial or not), putting the first stone to the general theory of relativity.

Thus, the idea that all systems should be equivalent *a priori* has been an efficient heuristic in the history of dynamics. But, noteworthily, this elementary principle is missing in QM, at least in the orthodox interpretation (as we shall see, standard QM is based on the distinction between classical apparata and quantal systems). In the light of past conceptual discoveries, this situation appears quite unsatisfactory. Can we reconcile QM with the monistic principle?

Outline of this memoir

This memoir consists in three parts. In part I, we analyze the origin of the problem of the interpretation of QM. As any scientific problem, it stems from the contradiction between two empirically successful conceptions of physical reality. The first one was championed by Einstein, as a synthesis of the teachings of pre-QM physics: we term this conception 'Einstein realism'. The second one is the content of the quantum revolution, which we propose to subsume under the expression 'Bohr's contextualism'. The opposition between these views led to the so-called 'hidden-variables program', surveyed at the end of part I. In part II, we present the main concern

[6]The upcoming reflexions are quite idiosyncratic, and clearly transcent the usual meaning of 'physical relationalism'. We present them as a personal motivation for the study of relational theories.

[7]More accurately, Mach's doctrine is called phenomenalism. The term operationalism came later, with the work of the philosopher Bridgman. Nevertheless, physicists usually use the latter term when referring to the necessity of operational definitions.

[8]The project to develop a Machian theory of dynamics, which would make no use of the idealized notion of 'reference frame', hence of space and time, was explored in some depth by Barbour (see [39]).

of this memoir, which is the Relational intepretation of Quantum Mechanics (RQM), and explore some of its insights, notably on the subject of the 'EPR paradox'. In part III, we present our (unfinished) investigation of the possibility to give RQM a formal support – something like a 'relational quantum logic'. The explicit motivation for this investigation is Rovelli's claim that RQM provides a successful scheme for *reconstructing* the formalism of QM. This reconstruction is the ultimate goal of the relational approach to QM.

Contents

Part I

Background: quantum mechanics and physical reality

Chapter 1

Einstein's realism

Einstein used to be a vehement opponent to the widespread opinion that QM qualifies as a 'fundamental theory'. His opposition to Bohr on this matter is famous and has been much commented, and it is not the purpose of this part to give a historically accurate account of this debate. Rather, our intention is to distinguish between two influential conceptions of fundamental physics, one encapsulating the core of classical physics ('Einstein realism'), the other one the novelty of quantum physics ('Bohr's contextualism'). This dialectic opposition will help us see the stake of the problem of the interpretation of QM.

According to the philosopher Fine, 'Einstein realism' relies on two stringent criteria: observer-independence, and causality [23]. Let us see what they consist in.

1.1 The observer-independent realm

The success of classical physics, including general relativity, strongly relies on an epistemological assumption called *realism.* The realistic doctrine supposes that the theoretical entities referred to in the scientific discourse are *faithful representations* of a *prestructured, external* physical reality. This credo, which was Einstein's, is at the core of the pre-QM scientific conception of the world. In his words:

> "Physics is an attempt conceptually to grasp reality as it is thought independently of its being observed." ([36], p. 81).

Thus, in the realistic viewpoint, physics is more than the austere *construction* of the comprehensive 'catalogue' of nomological regularities; it is the *description* of entities constituting the "observer-independent realm". Either taken literaly (metaphysical realism) or heuristically (motivational realism), the prejudice that the world is made of *objects*, carrying absolute *properties,* is rooted in the commonsensical behaviour of working physicists. For what are they actually studying, if not the "real state of affairs"?

The realistic stance has strong implications concerning the form of any fundamental physical theory. Its main feature, observer-independence, can be translated into the following constraints.

Hypothesis 1 *Value definiteness (VD): All observables defined for a quantum system have definite values at all times. More explicitely, there always exists a function, called a* valuation function V*, mapping any observable to a member of its spectrum, and representing the actual value of this observable.*

In a world where VD holds, the following statement is fully justified:

Corollary 1 *Counterfactual definiteness (CD): It is meaningful to speak of the definiteness of the outcome of a measurement, even if the latter is not actually performed.*

The other essential aspect of observer-independence is *non-contextuality*, which means that:

Hypothesis 2 *Non-contextuality. (NC) If a QM system possesses a property (value of an observable), then it does so independently of any measurement context, i.e. independently of how that value is eventually measured.*

1.2 Causality and space-time localization

In Einstein's mind, the 'observer-independent realm' can hardly be conceived, unless it is prestructured by *causality.* This principle demands that :

Hypothesis 3 *Causality: All fundamental laws should be deterministic. Accordingly, any fundamental theory should be free of probabilistic concepts.*

Furthermore, Einstein's realism relies on the *locality principle,* which roots the kinematical concepts of space and time in the very structure of physical reality:

Hypothesis 4 *Locality: Physical processes unfold in the spacetime continuum in such a way that distant objects cannot have instantaneous mutual influence.*

"Here" and "now" – spacetime localization – are primitive notions without which the very possibility to mentally divide the world in distinct objects is unavailable. Thus, it was one of Einstein's leitmotiv to find a spatio-temporal scheme underlying the quantum-mechanical algebra.

To summarize, Einstein realism is the claim that a fundamental physical theory consits in the *description* of objects *as they are independent of their being observed*, in a *causal* and *local* theoretical scheme.

Chapter 2

Bohr's contextualism

Quantum theory clearly does not fit the canons of Einstein realism. As far as empirical success is concerned, though, one can rightfully defend that QM contains as much 'truth' on the nature of physical realtity as Einstein realism. So, before we even try to reconcile these two, it is essential to be attentive to the lesson of QM. Let us listen to Bohr's understanding of this lesson, through his notion of *complementarity.*

2.1 Complementarity

The lessons of micro-experiments

The lesson Bohr took from the alternative wave-particule behaviour of matter in microphysics experiments is twofold:

- a physical experiment is codefined by the system under investigation *and* the actual physical context used to perform the measurement:

  > "[...] it is now highly necessary, in the definition of any phenomenon, to specify the conditions of its observation, the kind of apparatus determining the particular aspect of the phenomenon we wish to observe; and we have to face the fact that different conditions of observation may well be incompatible with each other to the extent indicated by indeterminacy relations of the Heisenberg type" [34].

- the measurement context can be described only in *classical* terms:

  > "However far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms." [37]

The combination of these two statements constitutes the *complementarity principle*, and is arguably the conceptual core of QM. In order to make full sense of this physical insight, we need to understand the meaning of 'classical' in the Bohrian lexicon.

In Bohr's account of the measurement process, an essential dichotomy has to be assumed: on the one side, is given a *quantum system*, which is the naturalized object of the description, subject to the laws of QM; on the other hand, the *classical* measuring apparatus, which is *not* described by the theory, but rather acts as a transcendental precondition of the physical description. Between them lies the so-called 'Heisenberg cut', which delimitates the frontier between the quantum and the classical domain.

The first theoretical function of the classical apparatus in QM is to determine a *perspective* through which the quantum system is envisaged. This persective determines the kind of behaviour, corpuscular or wave-like, that is accessible in the experiment. The second theoretical function of the classical apparatus is to provoke the *actualization* of accessible properties of the quantum system, as opposed to the variety of *potential* properties pertaining to the system. So to say, the Heisenberg cut acts as a boundary between potentiality and actuality. At the formal level, the transition between potentiality and actuality is represented by the so-called 'collapse' of the wavefunction. Finally, its third function is the *conceptual accessibility* of the system's properties: we cannot conceive other observables than the ones inherited from classical physics, like position or momentum. The apparatus is here to carry such classical properties.

The fact that different classical contexts can be alternatively, but not necessarily simultaneously, referredd to is the origin of the term 'complementarity'.

At this point, let us emphasize that complementarity is not to be confused *a priori* with the Heisenberg relations and their 'disturbance interpretation', to which we now turn.

The primitive "disturbance theory", and its refutation

During the period ranging from 1927 to 1929, Bohr developed a tentative explanation of complementarity, based on the notion of *irreducible disturbance*. His objective was to account for the necessity to include the measuring apparata in the description of quantum phenomena, within a reductionist framework. In addition, he expected his disturbance theory to give a mechanistic interpretation to the Heinsenberg inequalities.

Bohr postulated the existence of a "quantum of action", involved in all physical interactions. Because of its finite value, it was supposed to impose a limit to the divisibility of atomic processes, and hence to the accuracy of any measurement. It is precisely this limitation which is encapsulated by the Heisenberg relations.

The influence of this simplistic account of complementarity, which mixes up the statuses of complementarity and the Heisenberg relations, remains influential years after Bohr himself abandoned it, in the early 30's. Feynmann's vocabulary, for instance, is clearly reminiscent of it:

> "If an apparatus is capable of determining which hole the electron goes through, it *cannot* be so delicate that it does not disturb the pattern in an essential way. No one has ever found (or even though of) a way around the uncertainty principle."

In recent experiments [35], Englert, Walther and Scully eventually disproved Bohr's disturbance theory[1]. Indeed, they designed an experimental setup enforcing interferences without involving any exchange of momentum between the measuring apparatus and the system. What is more, they argue that the Heisenberg inequality plays stricly no role in the complementary behaviour of the investigated system.

The experimental setup consists in a two-slit interferometer, through which are sent excited atoms in a long-lived Rydberg state. Each slit is equipped with a micromaser cavity, *i.e.* a cavity filled with microwave radiation. Thanks to the resonant coupling of the excited atoms to the microwave field, the cavity serves as a which-way detector: each time an atom goes through the cavity, a desexcitation photon is emitted and recorded. The *tour de force* is of course that this detection has no influence on the center-of-mass motion of the atoms. This is because the measuring interaction involves internal degrees of freedom of the atoms only, so that the detector is effectively "so delicate that it does not disturb the interference pattern", as the authors ironically put it[2].

Although no momentum transfer can be invoked, the result of the experiment confirms Bohr's intuition: the presence of which-way detectors destroys the interference pattern. This conclusion shows that the complementarity principle is more fundamental than the uncertainty principle, and requires an abstract formulation which does not refer to the vague notion of irreducible disturbance.

Contextuality

At this point, we still have no clue about the answer to the primary question "What is QM about?". However, we have learned enough to formulate an answer to the secondary question: "What is QM?": *QM is a probabilistic predictive scheme, predicating the value of physical observables defined relative to mutually incompatible contexts.* This is the content of the inbuilt *contextuality* of QM.

[1] In fact, Bohr's disturbance theory was already ruled out by the EPR argument, as early as 1935 [28, 33, 23]. Nevertheless, with respect to the physical roots of complementarity, Englert, Walther and Scully's experiment is simpler and more suggestive. The EPR argument will be discussed hereafter.

[2] The reason why Feynmann could not think of such a setup is of course that, in the 50's, interference with "big" atoms rather than photons or electrons were considered highly impracticable.

Chapter 3

Hidden variables

In the face of Bohr's contextualism, it is hard to overcome a feeling of unease, due to its flat contradiction with Einstein realism. Since the quantum-mechanical empirical predictions are unquestionable, the realist's recourse is to take QM as an efficient, but *incomplete* description of physical reality. This is the starting point of the 'hidden variables program', which intends to achieve a reconciliation between Einstein realism and QM.

3.1 Quantum thermodynamics

The structuring analogy which has inspired the hidden-variables program relates QM to the history of thermodynamics. As well known, thermodynamical concepts such as heat or entropy, became truly understood only when Maxwell, Boltzmann, Gibbs and Einstein were able to connect them to a mechanistic (hence causal) picture. Within this framework, the use of probabilistic concepts is unproblematic: they account for the impracticability of the specification of all initial conditions for the dynamical system consisting of a macroscopic number of particles.

Undeniably, this application of the reductionist paradigm — the 'atomic hypothesis', as physicists of the nineteenth century used to call it — shed great light on the foundations of thermodynamics. In the context of QM, the temptation is great to recycle the heuristic, and postulate the existence of a more fundamental reality than that predicated by the usual formalism. According to this viewpoint, standard QM would be a coarse-grained version of a deterministic 'subquantum world', fitting the canons of Einstein realism. Of course, if this approach turned out to be tenable, the conceptual clarification would be considerable: QM would reduce to a refined version of statistical *classical mechanics*.

3.2 The general scheme

Definition 1 *We say that a quantum system with Hilbert space $\mathcal{H}$ admits* hidden variables *(HV) if there exists a measurable space (Λ, Σ) such that every state $\psi \in \mathcal{H}$ can be represented as a probability measure μ_ψ on Λ, and every observable A as a measurable map $\tilde{A} : \Lambda \to \mathbb{R}$, whose expectation value with respect to μ_ψ is consistent with quantum-mechanical predictions:*

$$\langle\psi| A |\psi\rangle = \int_\Lambda \tilde{A}\, d\mu_\psi$$

In such a scheme, the hidden variables $\lambda \in \Lambda$ are thought of as the subquantum extension of the classical phase (p, q) of Hamiltonian mechanics. In the spirit of Einstein causality, the evolution of λ is governed by some yet-to-be-discovered deterministic dynamical equations, of the usual form: $\frac{d\lambda}{dt} = f(\lambda)$. Clearly, this equation, together with the 'anti-correspondence' rules $A \to \tilde{A}$, implements VD. Indeed, one has the obvious valuation function which, given the hidden state λ of the system, maps each observable to its value:

$$V_\lambda : A \longmapsto V_\lambda(A) = \tilde{A}(\lambda) \tag{3.1}$$

Counterfactual definiteness and the sample space Λ

Thus, as required by the realistic credo, VD is at the core of the HV scheme: at each time, any observable A has a definite value, $\tilde{A}(\lambda)$. But another, subtler, feature of HV theories is their reliance on CD. Let us analyze this point in some detail, for it will be essential to the understanding of Bell's theorem (see below).

We have seen that the key ingredient of the HV program is the existence of a sample space Λ. Now, what we want to emphasize here is that its probabilistic interpretation is based really on CD, and not on VD (but recall that VD$\Rightarrow$CD). Indeed, in probability theory, the elementary events (the λ's) are not realized in actuality, otherwise the recourse to probabilities would be pointless. For instance, when I play lotto, I do not assume that all possible outcomes are actually realized, but only that among all possible outcomes, one is actually realized. The role of the sample space is therefore not to describe the actual situation, which can be inaccessible, but rather *to exhaust all potential situations*. In other words, the implicit reasoning underlying the recourse to the space Λ in HV theories is: I may ignore what is actually happening, but I certainly know what could happen, and can rightfully act *as if one of the potential events was actual*. In this sense, the existence of the space Λ is the translation CD.

Thus, to our understanding, the most essential feature of HV theories is really CD. If, for some reason, VD was ruled out, HV partisans could still rely on CD. They could say: "OK, the world is not really as we picture it, but appears to be structured *as if it were*." They would thus adopt the CD-based structural realism instead of their original VD-based metaphysical realism. And it would be no big deal.

An example: de Broglie-Bohm theory

The most famous formulation of QM in terms of HV is the so-called de Broglie-Bohm theory [55]. Here, elementary quantum objects are pictured as point-particles moving in physical space. The hidden variables are just their spatial positions, so that for a N-particles system, $\Lambda = \mathbb{R}^{3N}$. Given a quantum state $\psi \in \mathcal{H} = L^2(\Lambda)$, the Born rule provides the corresponding probability measure on Λ: $\mu_\psi = |\psi|^2 \, d^{3N}x$. The dynamics of the i-th particle (of mass m_i) is given by the equation $m_i \frac{dx_i}{dt} = \nabla_{x_i} S$, where S is the phase of the wave function: $\psi = |\psi| \, e^{iS}$. Of course, the wave function ψ itself evolves in time according to the Schrödinger equation $i\hbar\partial_t\psi = H\psi$.

It can been shown that the experimental predictions provided by the de Broglie-Bohm theory are consistent with QM. This fact proves that the hidden variables program is not unviable *a priori*. Still, does the de broglie-Bohm theory achieve the sought-for reconciliation between QM and Einstein realism? Clearly not: its realization of VD is purely formal, since the hidden variables remain inaccessible to observation. Even worse, it is both non-causal (because of the postulated Born rule) and non-local (because of the non-local right-hand side of the dynamical equation $\nabla_{x_i} S(x_1, ..., x_N)$). In fact, we shall soon see that there are severe theoretical obstructions to the implementation of Einstein realism. These are the so-called 'no-go' theorems.

3.3 No-go theorems

No-go theorems are attempts to show, on the basis of QM itself, that the HV program is vain. This is done by expliciting 'crippling' constraints on the hidden variables. The most significant results in this direction are von Neumann's ([38], 1932), Gleason's ([45], 1957), Kochen and Specker's ([15], 1967) and foremost Bell's ([14], 1964) theorems.

The von Neumann-Gleason-Kochen-Specker argument

von Neumann was the first to prove a no-HV theorem, in his early 1932 treatise [38]. His argument, which appeared to be operationally flawed, was completed by Gleason 25 years later, in a celebrated theorem. A third step was taken in 67 by Kochen and Specker, strengthening von Neumann's and Gleason's results. Because of their identical conclusions, we shall, for the sake of simplicity, disregard the details of their reasoning (which are somewhat different) and identify their results as the 'vNGKS' argument[1].

Although its explicit intention was to purely and simply refute the possibility of a HV solution to the problem of quantumness, the vNGKS argument proved something else[2]. Namely, it proved that, for a HV theory to be compatible with QM's predictions, it has to be *contextual*. This means that the only kind of VD which is consistent with the quantum formalism violates the second assumption of Einstein realism, non-contextuality. To understand this argument, we need to recall the standard quantum-mechanical definition of a *measurement context*.

Definition 2 *A* measurement context *for an observable A is a set of commuting observables $\mathcal{C}$ such that $A \in \mathcal{C}$.*

Thus, the vNGKS argument is that, in general, there does not exist a valuation map V defined on the set of quantum observables, which is both independent of the measurement context and consistent with the Born rule. How did vNGKS manage to prove such a statement?

[1] For the meticulous reader, von Neumann proved that no value map V_λ can be additive on self-adjoint operators, in contradiction with the expected linearity of the expectation value map. Because the sum of non-commuting operators is operationally ill-defined, his result was not conclusive. Gleason proved the same result, under the weakened hypothesis of additivity on commuting projectors. Finally, the Kochen-Specker theorem involves only a finite number of observables, in contrast with the continuum of projectors required by Gleason's argument. Noteworthily, it was the realist Bell who gave to these characters most of their physical arguments. See [43].

[2] Years after Bohm's publication of his HV theory, it was still very often argued that HV was incompatible with QM. This statement is obviously wrong, since the de Broglie-Bohm *is* compatible with QM. Again, see [43].

Following the outline given in [44], we notice that assigning a value to an observable O, with spectral projections $\{P_i\}$, is equivalent to distinguishing exactly one member of the set $\{P_i\}$, and assigning it the value '1', while assigning all the other projections in $\{P_i\}$ value '0'. Let $\mathcal{P}$ denote a set of projections, and $\overline{\mathcal{P}}$ denote the set of all observables whose spectral projections lie in $\mathcal{P}$. Then, whether there can be hidden variables that uniquely determine the measured values of all the observables in $\overline{\mathcal{P}}$ is equivalent to asking whether there exists a *truth function* $t : \mathcal{P} \to \{0,1\}$ satisfying:

$$\sum_i t(P_i) = 1 \text{ whenever } \sum_i P_i = I \text{ and } \{P_i\} \subseteq \mathcal{P}. \tag{3.2}$$

Now, the vNGKS arguments relies on the following theorem:

Theorem 1 *For a QM system whose Hilbert space's dimension is greater than* 3*, there always exists a set of observables* $\overline{\mathcal{P}}$ *such that* $\mathcal{P}$ *admits no truth function.*

In fact, for such a truth function to exist and be well-defined, it has to depend on the whole resolution of the identity $\{P_i\}$, and not only on the projections themselves. Because of the 1-to-1 correspondence between resolutions of the identity and sets of commuting observables, this amounts precisely to contextuality.

Thus, in a consistent HV theory, the hidden state λ has to be completed by the specification of the actual measurement context in order to fix the value of the observables[3]. Needless to say, this is hardly compatible with the initial motivation of the HV program, which was to free QM from holistic, measurement-like influences. This is the first serious obstruction to a convincing HV theory and, simultaneously, a very strong confirmation of Bohr's intuition on contextualism.

Bell's theorem

But there is worse. With the vNGKS argument, we have discovered that the kind of VD tolerated by QM is severely constrained (to contextual VD). But, at this point, CD remains unaffected. The strength of Bell's theorem [14] lies in its addressing this subtler structural feature of HV theories, showing CD's incompatibility with *locality*. Here is an adaptation of Bell's reasoning (inspired from d'Espagnat, [46]), which emphasizes its reliance on the sole CD.

Suppose that the HV's are local, and consider three quantities A_1, A_2 and A_3, associated with two identical but space-like-separated systems α and β. Bell's (apparently innocuous) observation is the following: *If the A_i's have definite values, and if these values are somehow correlated, then this correlation is constrained by quantitative inequalities*[4]. Let us see how.

As explained in paragraph 2.3, CD induces a set-theoretic model for the possible experimental events: each (counterfactually determined) hidden state of α (resp. β) defines an element of a sample space Λ_α (resp. Λ_β), and the complete experiment is described by the Cartesian product $\Lambda = \Lambda_\alpha \times \Lambda_\beta$. Assume for simplicity that the A_i's can take only two values, say '0' and '1'.

Let us focus, for a moment, on one system, say α. We have a natural partition of Λ_α in 8 subsets, corresponding to the values σ_i's of the A_i's. Let us denote $n(\sigma_1, \sigma_2, \sigma_3)$ the cardinal of an element of this partition, and N the cardinal of Λ_α. Let us finally define the correlation $M(i,j)$ between A_i and A_j by:

$$M(i,j) = N^{-1} \sum_{(\sigma_1,\sigma_2,\sigma_3)\in\{0,1\}^3} \sigma_i \sigma_j n(\sigma_1, \sigma_2, \sigma_3) \tag{3.3}$$

Using the fact that $\sigma_i \in \{0,1\}$, it is easy to show that, for $i \neq j \neq k$:

$$|M(i,j) - M(j,k)| \leq 1 - M(k,i) \tag{3.4}$$

Now, suppose the A_i's are initially anti-correlated between α and β : $\sigma_i^\alpha = -\sigma_i^\beta$. Such a situation is commonly encountered in QM (see the paragraph on the EPR argument, below). *Because of locality*, this anti-correlation is not affected by the measurements performed on the particle individually. Defining the correlation function between α and β by:

$$P(i,j) = N^{-1} \sum_{(\sigma_1,\sigma_2,\sigma_3)\in\{0,1\}^3} \sigma_i^\alpha \sigma_j^\beta n(\sigma_1, \sigma_2, \sigma_3) \tag{3.5}$$

the anti-correlation entails:

[3] This is the case, in particular, of the de Broglie-Bohm theory: there, the motion of the hidden particles is determined by the wave-function, which of course depends holistically on the measuring environment.

[4] Note that this statement is true *irrespective of the actual value of the A_i's.*

$$P(i,j) = -M(i,j) \tag{3.6}$$

and hence for $i \neq j \neq k$:

$$|P(i,j) - P(j,k)| \leq 1 + P(k,i) \tag{3.7}$$

This is one of the so-called 'Bell inequalities'. Since QM violates this inequality, as can be seen by taking as A_i's spin observables ($[A_i, A_j] = i\epsilon_{ijk}A_k$), we conclude that, to be consistent with QM, CD has to contradict locality. This is Bell's theorem. Importantly, the violation of Bell's inequalities was experimentally confirmed by Aspect *et al.* [1, 2] in the early 80's.

So, is realism compatible with QM?

In this section, no-go theorems have taught us that for a HV theory to qualify as a 'completion' of QM, it has to be both *contextual* and *non-local*. What is more, we have seen that one of these features – non-locality – acts at a very deep level, namely at the level of counterfactual definiteness. How can we interpret these results?

To answer this question, let us recall that the HV program sprang from Einstein realism. As we have tentatively shown, Einstein realism is in no way reducible to the sole requirement of value definiteness. Rather, it encapsulates in a realistic epistemological scheme the most fundamental principles or heuristics of classical physics. Unfortunately, these insights, when implemented in HV theories, appear to contradict QM. Eventually, the price to pay to maintain strict realism at the foundation of QM amounts to contradicting Einstein realism itself, through locality or non-contextuality. This situation can hardly be considered satisfactory. Contrary to its primary intention, HV-based realistic QM *does not* extend the domain of classical physics. On the contrary, it spawns highly non-classical physics, making even more obscure the connection between QM and pre-QM physics.

This said, we do not believe that realism is incompatible with QM, as is sometimes argued. In fact, we think that such a statement would be a categorial mistake, because realism is not the kind of commitment that can be confronted to empirical data (and QM is nothing but empirical data). Rather, we feel that the realistic HV-like scheme fails to provide a consistent and unified picture of the world. In particular, it fails to escape Bohr's contextualism, and does not rehabilitate Einstein's causality.

Part II

Relational quantum mechanics

Chapter 4

Rovelli's relational interpretation of quantum mechanics

In this chapter, we introduce Rovelli's relational quantum mechanics (RQM), essentially as presented in the original 1996 paper [5]. Here, we see how quantum theory and its key feature – contextuality – naturally fits, and even extends, the relational scheme proposed by Leibniz, Mach and Einstein. Far from promoting an arbitrary interpretational move, we argue with Rovelli that it is QM *itself* which demands us to acknowledge the relational content of its predictions. As suggested already, the picture of reality emerging from RQM departures from Einstein realism in a rather radical way. But, according to Rovelli [6]:

> "If we want to understand nature, our task is not to frame nature into our philosophical prejudices, but rather to learn how to adjust our philosophical prejudices to what we learn from nature."

So, Rovelli, what are we to learn from nature this time?

4.1 A fact of quantum mechanics

QM itself drives us to the relational perspective. This is a strong claim, which needs to be precisely justified. In order to achieve this purpose, Rovelli's first step is an original analysis of the process of measurement, entitled the 'third person problem'.

The third person problem

Consider a quantum system S. Assume for simplicity that S is a qubit, *i.e.* that S is completely described by a two-valued observable A, with eigenstates $|0\rangle_S$ and $|1\rangle_S$ and eigenvalues a_0 and a_1 respectively. Let O be an apparatus measuring A on S at time t (with pointer states $|0\rangle_O$ and $|1\rangle_O$), and P another apparatus measuring the state of the couple $S+O$ at time $t^{'} > t$. What is the standard quantum-mechanical description of this (indeed banal) situation?

O's viewpoint

When described from the point of view of O, at time t, the state of S – generically $|\psi\rangle_S = \alpha\,|0\rangle_S + \beta\,|1\rangle_S$ – undergoes one of the following transformations:

$$|\psi\rangle_S \rightarrow \begin{cases} |0\rangle_S \,\text{with probability}\, |\alpha|^2 \\ |1\rangle_S \,\text{with probability}\, |\beta|^2 \end{cases} \tag{4.1}$$

Of the two potential values of A, one is actualized at t: a_1 in the first case, a_2 in the second case. This is basic QM, namely von Neumann's projection postulate.

P's viewpoint

From the point of view of P, the previous measurement is described as follows. Denoting $|init\rangle_O$ the initial state of O, the couple $S+O$ undergoes the following evolution:

$$|\psi\rangle_S\,|init\rangle_O \rightarrow 2^{-1/2}(\alpha\,|0\rangle_S\,|0\rangle_O + \beta\,|1\rangle_S\,|1\rangle_O) \tag{4.2}$$

Hence, at time t, the value of A is undetermined: it is potentially a_1, with probability $|\alpha|^2$, and potentially a_2, with probability $|\beta|^2$[1]. This follows from the linearity of the Schrodinger equation for the couple $S+O$.

[1]The probabilities refer to P's subsequent measurement, performed at time $t^{'}$.

The main observation

Now for the main observation: QM does not give a consistent answer to the question "does A have a definite value at time t?". Indeed, according to the previous analysis, the answer is twofold and apparently self-contradictory:

- Relative to O, the answer is *yes* (and O is able to tell *which* value)
- Relative to P, the answer is *no*

Where does this ambiguity come from? Have we somehow misused the quantum-mechanical formalism? Obviously not. QM, as it stands, gives us no alternative but to acknowledge with Rovelli that [5]:

> "In quantum mechanics different observers may give different accounts of the same sequence of events."

We emphasize that, at this point, no extra-interpretation is involved. We are facing a quantum-mechanical *fact*[2]. And, true, a problematic fact, given the standard view that nature cannot be so schizophrenic to allow such a dualistic picture of the 'real state of affairs'. Or can it?

Common reactions

At this point, it is interesting to review a couple of widespread reactions to the third person problem. This is done extensively in Rovelli's article [5], and we shall here restrict ourself to the two most influential ones. In particular, we shall disregard the views which question the validity of QM's predictions[3].

The Einsteinian incompleteness complaint

According to the main canon of Einstein realism, a question referring to the 'real state of affairs' can be, and *must be*, given a clear-cut answer. In particular, it is the *raison d'etre* of fundamental physics to unambiguously predicate about the actual properties of actual systems. Therefore, the third person problem, in which QM fails to answer the question "What is the value of the observable A at time t?", is a rightful concern, and allows no other conclusion than the following: QM is an *incomplete* description of reality.

Unfortunately, we have seen that, because of no-go theorems, there cannot be any completion of QM consistent with both its empirical predictions and full Einstein realism. The only possible escape for the realist is to abandon essential features of Einstein realism, such as causality, and get used to the idea that, even though S's real state of affairs is definite, something irrepressibly prevents him from knowing it[4]. But doesn't QM deserve better than a shrug of resignation[5]?

Opposite to the incompleteness complaint, RQM's stance can be subsumed in the following postulate [5]:

Postulate 1 *QM provides a complete and self-consistent scheme of description of the physical world, appropriate to our present level of experimental observations.*

So, against Einstein and the realists, we assume that QM is complete.

The Bohrian canonical cut

On the other side of the interpretational rift, Bohr's explanation of the third person problem goes as follows: the paradox comes from the illegitimate application of the quantum-mechanical formalism to the observer O.

Indeed, recall that Bohr's interpretation relies strongly on the Heisenberg cut, which, in his view, is to be though of as a definite boundary between a quantum system and a classical apparatus. In the scenario we are discussing, O, being a measuring apparatus with respect to S, is classical. It is therefore pointless, and even meaningless, to apply the Schrodinger equation to the couple $S + O$. Thus Bohr dissolves the third person problem.

But it is of course very hard to understand how objects like measuring apparata, which are believed to made of atoms and nothing else, could be intrinsically classical. If QM applies to atoms, and if it can describe composite systems as well as simple ones, then it should apply to O as well. Bohr, needless to say, was aware

[2]In fact, this observation has be constantly revisited since von Neumann's 1932 treatise, notably by Wigner (this is why P is sometimes called 'Wigner's friend'). It is the core of the 'measurement problem'

[3]Recall that, to this day, no experiment has ever disproved any quantum-mechanical prediction.

[4]This solution is advocated for instance by the Bohmian realist school. But because of its resigned departure from causality and locality, Einstein himself showed little interest to this interpretation.

[5]In the realist line of thought, a noteworthy attempt to overpass the apparent conspiracy of the real-but-hidden state of affairs is made by A. Valentini, in a program termed 'quantum non-equilibrium'. See for instance [42]

of this difficulty. His answer to it was that, *for all practical purpose*, it is sufficient to reason *as if* measuring instruments were intrinsically classical. The key idea is that QM is not, and *needs not be*, universally valid.

This view was fully coherent in Bohr's times, when the Heisenberg cut was in all practical situations clearly defined. Quantum systems were micro-objects (like electrons and such), and measuring instruments were macro-object (like Stern-Gerlach apparata), so there was no conceivable confusion between the system and the observer. The Heisenberg cut was, so to say, canonically given. But this is not the case any-longer. Examples of mesoscopic quantum systems are common, and, in situation were the environment is taken into account, the definition of the observer becomes quite ambiguous. For contemporary experimenters, Bohr's distinction, according to which S is quantum and O is classical, would sound arbitrary. Their everyday practice heavily relies on the fundamental freedom of QM to move the Heisenberg cut *ad libitum*.

It is therefore essential to affirm against Bohr, and with Rovelli, that [5]:

Postulate 2 *All systems are equivalent. Nothing distinguishes a priori macroscopic systems from quantum systems. If the observer O can give a quantum description of the system S, then it is also legitimate for an observer P to give a quantum description of the system formed by the observer O.*

Notice that this principle complies with the monistic desideratum, previously mentioned. We feel that the reconciliation of QM with this principle, if it eventually turns out to be tenable, is good news!

4.2 Relativity of actuality

The relational interpretation of QM is now in front of us. QM allows different observers to give different accounts of the same sequence of events (main observation); nevertheless, QM provides a complete description of this sequence of events (Postulate 1); the discrepancy between the observers' descriptions cannot be attributed to any *a priori* distinction between them (Postulate 2). This triad of statements resolves itself into one: *each quantum mechanical description has to be understood as relative to a particular observer*.

The claim of the relational approach is thus that a number of confusing puzzles raised by QM result from the unjustified use of the notion of objective, or absolute, state of a physical system. Just like the question "what is the velocity of S?" is ill-posed unless we specify a reference frame, the question "In the $S-O-P$ scenario, what is the value of A for S at time t?" is ill-posed. As we already put forward when discussing the third person problem, the answer depends on the viewpoint. *Relative to O*, the value of A at t is definite and actual; *relative to P*, the value of A at t is indefinite.

Let us emphasize here that the relativity of properties in RQM is really an extension of usual relativity: as the former does with velocity, it states that physical quantities always pertain to pairs of systems. The value of A at time t is a property of the couple $S-O$, and not of the sole S. So to say, RQM relativity is Einsteinian relativity[6], generalized to all observable quantities. But because in QM some quantities happen to be indefinite, RQM adds a twist to pre-quantum relativity: not only the value of the same observable A, but also its very definiteness, can differ from one observer to another.

RQM and physical reality

As opposed to the realistic, HV-based approach, RQM departs explicitly from Einstein realism. Here, the structure of the theory, manifestly reluctant to fit Einstein's metaphysical canons, is taken really seriously.

In RQM, physical reality is taken to be formed by the individual *quantum events* (facts[7]) through which interacting systems (objects[8]) affect one another.

Quantum events exist only in interactions[9] and the reality of each quantum event is only relative to the system involved in the interaction. In particular, the reality of the properties of any given system S is only relative to a physical systems O that interacts with S and is affected by these properties. If O keeps track of the properties of S (relative to O), then O has information about S. According to RQM, this information exhausts the content of any observer's description of the physical world.

[6] Here, we are following the Machian line of thought which led Einstein to special relativity: as is the case for absolute simultaneity, there is no operational definition of observer-independent comparison (one is tempted to say "synchronization") of different observers' information about a system. The information of different observers can be compared only by a physical exchange of information between the observers. But since all systems are quantum systems, any exchange of information is itself a physical interaction, and as such subject to the laws (and in particular the uncertainties) of quantum mechanics. The comparison of information is itself a physical quantum process. Thus, the usual prejudice that it is meaningful to speak of the 'state' of a system, irrespective of a particular observer, is pointed out by RQM as unphysical, in the Machian/Einsteinian sense.

[7] "1.1 The world is the totality of facts, not of things" [24].

[8] "2.01 An atomic fact is a combination of objects (entities, things). 2.011 It is essential to a thing that it can be a constituent part of an atomic fact" [24].

[9] "2.0121 There is no object that we can imagine excluded from the possibility of combining with others" [24].

Michel Bitbol proposes to qualify this approach as a *meta-description* [10]: RQM is the set of rules specifying the form of any such physical description. In that sense, RQM is faithful to Bohr's epistemological position, as presented for instance in [25]:

> *It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature.*

Still, RQM adds an essential twist to this position of Bohr: For Bohr, the "we" that can say something about nature is a preferred macroscopic classical observer that escapes the laws of quantum theory: facts, namely results of quantum measurements, are produced interacting with this classical observer. In RQM, the preferred observer is abandoned. Indeed, it is a fundamental assumption of this approach that nothing distinguishes *a priori* systems and observers: any physical system provides a potential observer, and physics concerns what can be said about nature on the basis of the information that *any* physical system can, in principle, have. The preferred Copenhagen observer is relativized into the multiplicity of observers, formed by *all* possible physical systems, and therefore it does not anymore escape the laws of quantum mechanics.

Different observers can of course exchange information, but we must not forget that such information exchange is itself a quantum mechanical interaction. An exchange of information is therefore a quantum measurement performed by one observing system upon another observing system.

The physical meaning of the wave function

Regularities in natural phenomena, that is, laws of nature, mean that an observer O may be able to make (probabilistic) predictions about future interactions with a system S on the basis of the information previously acquired via past interactions. The tool for doing this is the quantum state ψ of S.

Since predictions are based on past interactions, ψ is, first of all, essentially just a coding of the outcome of these previous interactions. But these outcomes are facts that are real only with respect to O. Therefore the state ψ is only relative to O: *ψ is the coding of the information that O has about S.* Because of this irreducible epistemic character, ψ is a relative state, and cannot be taken to be an objective property of the single system S, independent from O. Every state of quantum theory is a relative state.[10]

On the other hand, the state ψ is a tool that can be used by O to *predict* future outcomes of interactions between S and O. In general these predictions depend on the time t at which the interaction will take place. In the Schrödinger picture this time dependence is coded into a time evolution of the state ψ itself. In this picture, there are therefore two distinct manners in which ψ can evolve: (i) in a discrete way, when S and O interact, in order for the information to be adjusted, and (ii) in a continuous way, to reflect the time dependence of the probabilistic relation between past and future events.

Incidentally, notice that from the relational perspective the Heisenberg picture appears far more natural: ψ codes the information that can be extracted from past interactions and has no explicit dependence on time; it is adjusted only as a result of an interaction, namely as a result of a new quantum event relative to the observer. If physical reality is the set of these bipartite interactions, and nothing else, our description of dynamics by means of relative states should better mirror this fact: discrete changes of the relative state, when information is updated, and nothing else. What evolves with time are the operators, whose expectation values code the time-dependent probabilities that can be computed on the basis of the past quantum events.[11]

To see the full coherence between the epistemic interpretation of ψ and RQM's relativity of actuality, a re-discussion of the qubit example might be useful. Consider the case where S is a spin-1/2 particle, $A = \sigma_z$ is the z-component observable, with eigenstates $|\uparrow\rangle_S$ and $|\downarrow\rangle_S$. Suppose that, at time t, O measures σ_z and gets the outcome $\uparrow$. At same instant, P describes the state of $S+O$ by the superposition $2^{-1/2}(\alpha\,|\uparrow\rangle_S\,|\uparrow\rangle_O+\beta\,|\downarrow\rangle_S\,|\downarrow\rangle_O)$. Informally, P does not 'know' the z-component of S's spin. But this should not surprise us: σ_z is a relational quantity, which is defined relative to a z direction − which is, in this case, materialized by O. So, until P measures the spin of O, P does not even 'know' what the z direction is! It is only when the latter is 'known' that σ_z becomes an actual property of S. Again: *like any other quantity, σ_z is relational in nature.*

Quantum contextuality uncovered

Let us get back to Bohr's original insight: QM's irreducible context-dependence. Within the relational perspective, this contextuality is recovered most straightforwardly. It is precisely the key idea of RQM: quantum

[10] From this perspective, probability needs clearly to be interpreted subjectively. On a Bayesian approach to QM, see [18, 26]; for a more general defense of Bayesian probabilities in science, and a discussion of the relevance of this point for the EPR debate, see [19].

[11] This was also Dirac's opinion: in the first edition of his celebrated book on quantum mechanics, Dirac uses Heisenberg states (he calls them relativistic) [27]. In later editions, he switches to Schrödinger states, explaining in the preface that it is easier to calculate with these, but it is "a pity" to give up Heisenberg states, which are more fundamental. In what was perhaps his last public seminar, in Sicily, Dirac used a single transparency, with just one sentence: "The Heisenberg picture is the right one".

properties are defined only relative to an observing system O. The physical context, traditionally represented as a set of commuting observables, is thus simply the set of O's pointer variables. Bohr's mysterious holism is uncovered as the reduction of reality to correlation between systems. Complementarity is just the archaic formulation of RQM's relativity: there is no 'real state of affairs' to be unveiled by physical measurements. Each observer, through its own pointer variables, acts as a particular 'window' on the system. Different observers provide different windows. These windows need not be simultaneously accessible. Observing systems are just what Bohr explained: complementary windows.

4.3 von Neumann's consistency

So far, all we have said is that all quantum-mechanical quantities are relational. Their values have definite values only relative to well-defined observers. In such a fragmented description of physical reality, one question should arise: what about the unity of the world? If everyone can assign a different value to the same quantity, how do we not experience the most dismal chaos?

The question is relevant, and the answer reassuring: *The consistency of different observers' account of the same sequence of events is automatically ensured by the formalism of QM itself.* It was von Neumann who first put the emphasis on this aspect of the quantum theory of measurement (whence the name 'von Neumann's consistency'), namely on the freedom left by QM to freely move the Heisenberg cut without any predictive consequence. How is this consistency implemented in RQM? To see this, let us return to the third person scenario previously discussed.

At time t, O has measured the state of S through the observable A, and has obtained the value, say, 0. Thus, the state of S relative to O is $|\psi\rangle_S^{(O)} = |0\rangle$. At the same time t, relative to P, the state of the compound system $S+O$ is given by $|\psi\rangle_{S+O}^{(P)} = 2^{-1/2}(\alpha\,|0\rangle_S\,|0\rangle_O + \beta\,|1\rangle_S\,|1\rangle_O)$. Therefore, the subsequent measurement, performed at time t' by P on $S+O$, has two possible outcomes: $|0\rangle_S\,|0\rangle_O$ and $|1\rangle_S\,|1\rangle_O$. In both cases, *relative to P, the state of O and the state of S are consistent.*

This inbuilt consensuality of different observers' account of a given observation (here, the state of S), is taken in RQM to be *the most fundamental characteristic of the quantum formalism.* In the perspective of an axiomatization of RQM on first principles, in order to *derive* the quantum formalism, von Neumann's consistency will be an essential ingredient. We shall get back to this point later.

4.4 A metadescription of reality

Let us summarize the teaching of RQM. This interpretations discards the notion of observer-independent state of a system, and argues that all physical properties are defined *relative to some observer.* As shown by the third person problem, this view is strongly suggested by the quantum formalism itself, which attributes different by equally justified states to the same system, depending on the viewpoint envisaged. This fragmented, perspectival, account of physical reality is taken to be *complete.*

Thus, the referent of the RQM discourse is not some 'real state of affairs' of the Einsteinian style. RQM does not deal with 'objective properties', and as such cannot be termed a *description* of physical reality. Then what is it? Returning to the initial question: What is RQM about?

We have already given the answer to the his question. It was clearly pointed by Bitbol [10], and after him by van Fraassen [11]. RQM is a *metadescription of physical reality.* This means that its referent are the possible descriptions given by various observers, descriptions which are themselves made of quantum events, through which quantum properties are (relatively) actualized. RQM predicates the general form of such descriptions, and their predictive scope. And certainly *not* their hypostatic substance. This somewhat transcendental point of view was made clear by van Fraassen [11]:

> "There is no implication of possible specific information about what there is which is independent of any point of view; but there can be knowledge of the form that any such information, relative to particular vantage point, mast take."

With this perspective, new light is shed on the lack of determinacy of QM. Indeed, once acknowledged the indexical nature of any physical statement, the question arises: determinacy relative to whom? It appears that the complete determinacy advocated by Einstein realism implicitly relies on the existence of a purely external index, some kind of God capable of seeing the world 'from the outside'. In RQM, the scientific relevance of this type of panoramic viewpoint on the world is denied. Or, to be more precise, its indexical function is displaced. Formerly envisaged as describing the totality from the outside, this God is, in RQM, relativized to the multiplicity of *partial* observing viewpoints. This new God is nothing but the meta-observer of RQM, that is, the 'he' that makes quantum-mechanical predictions. Bitbol says this better than ourselves ([50], p. 299):

“[...] a piece of knowledge can be reasonably objective only it renounces being exhaustively objectifying; it can be reasonably exhaustive only if it is partly *participative*. Renouncing exhaustivity on behalf of a static and universal conception of objectivity constitutes a respectable option (which has showed its efficiency at the time of classical physics), but which leads, one day or another, to a manifestation of the epistemic “return of the repressed”. It is this, among other things, that quantum mechanics has taught us, in a sibylline though insistent manner. Losing sight of it again would contribute in no way to the progress of the thought.”

Chapter 5

A relational view on quantum statistics

5.1 The origin of indeterminacy: distinct observers

We have suggested above that complementarity springs from the necessity to relate certain observables to *distinct observers*. This fact was misted up in the standard interpretation of QM, because the relational nature of physical quantities was obfuscated by the *a priori* choice of a fixed, canonical observer. Only RQM offers us the possibility to acknowledge the origin of complementarity, and thus of indeterminacy. In this section, we tentatively argue that the necessity to take distinct observers into account stems naturally from the relational nature of quantum-mechanical physical quantities. In this respect, Dickson's discussion of quantum reference frames [49], to which we shall now turn, is very insightful.

Dickson's analysis of quantum frames and uncertainty

In the recent [49], Dickson proposes an original interpretation of the position/momentum uncertainty relation. This interpretation is based on a refreshing reminder on the foundations of dynamics. In non-relativistic mechanics (including QM), the formulation of dynamical laws requires the notion of *inertial frames*. These, in turn, are defined as Galilean transforms of the Newtonian pseudo-entity called 'absolute space', or more modernly, 'the background'. The latter, though empirically inaccessible, acts as a conceptual guarantor of the very existence of inertial frames. Since the dynamical equations are valid only in such inertial frames, it is a fundamental issue to have at hand a definite procedure for checking the inertially of a frame $\mathcal{R}$. The procedure in question is of course the recourse to a second inertial frame $\mathcal{R}'$, in which the constancy of $\mathcal{R}$'s momentum is tested. Notice that this procedure is crucially based on the relational nature of the momentum of $\mathcal{R}$: were the latter not relational, the shift to the viewpoint of $\mathcal{R}'$ would be pointless.

As long as the external viewpoint of $\mathcal{R}'$ is disregarded, Dickson argues, the position/momentum uncertainty relations which affect the systems observed in $\mathcal{R}$ remain hardly understandable. Its is only when one takes full account of the relational character of these quantities, hence when one switches to $\mathcal{R}'$, that things get clearer. Here is how.

Suppose that, in $\mathcal{R}$, the position Q_S of a system S is measured by a dedicated apparatus A (with momentum P_A). The interaction Hamiltonian can be taken to be $H_{int} = g(t)Q_S P_A$, where $g(t)$ is some smooth, peaked function of time. After a sufficient time, the effect of H_{int} is to correlate the position of S, Q_S, with the position of A, Q_A. Now, because of the Heisenberg relation, the momentum of S is irreducibly spread by this measurement:

$$\Delta Q_S \Delta P_S \geq \hbar/2. \tag{5.1}$$

Whence this uncertainty? Let us switch to the external reference frame $\mathcal{R}'$. In the passage from $\mathcal{R}$ to $\mathcal{R}'$, all observables are transformed according to the usual transformation law:

$$O \to O' = UOU^{-1}. \tag{5.2}$$

In this expression, $U = e^{-i(P'_S + P'_A)Q'_\mathcal{R}}$ is the representor of the Galilean transformation relating $\mathcal{R}$ and $\mathcal{R}'$, taken to be a pure translation for simplicity. Thus, for instance,

$$UQ_S U^{-1} = Q'_S - Q'_\mathcal{R}. \tag{5.3}$$

Now for Dickson's main observation. In the new reference frame $\mathcal{R}'$, it can be shown, using this transformation law, that:

$$i\hbar \frac{dP'_\mathcal{R}}{dt} = [P'_\mathcal{R}, H'] \neq 0 \tag{5.4}$$

In other words, the momentum of $\mathcal{R}$ itself is not conserved during the measurement, *even though there is no explicit interaction with* $\mathcal{R}$. Since P_S is defined relative to $\mathcal{R}$, and the momentum of $\mathcal{R}$ (relative to $\mathcal{R}'$) is 'disturbed' by the measurement of Q_S, P_S is intrinsically ill-defined during this measurement. Whence the uncertainty relations. (This 'whence' is of course very informal: we did not derive the uncertainty relations, but rather interpreted them.) Here is Dickson's conclusion on this matter [49]:

> "The point [of this paper] is to begin to understand the uncertainty relations as arising naturally from the peculiar way in which quantum theory embodies the fact that position and momentum (at least *as measured*) are relational quantities. As such they are well defined only when that relative to which they are defined, the 'reference body', is suitable for playing the role of defining them. As we have seen, there is good reason to think that a reference body that is used to define position is not, during the measurement of position, suitable to play the role of simultaneously defining momentum. [...] The same may be true of other relational quantities."

The lesson we take from Dickson's discussion is twofold: first, Rovelli's teaching that much of QM's mystery is hidden in the arbitrary fixing of the Heisenberg cut (here, between $S+A$ and $\mathcal{R}$) seems to confirm its potential; second, that the necessity to take into account distinct observers' viewpoints (here, $\mathcal{R}$ and $\mathcal{R}'$) when relational quantities are involved has to do with the 'irreducible quantum indeterminacy'.

Indeterminism and relational quantities

In somewhat different perspectives, a handful of authors have proposed to relate the 'objective indeterminism' of QM with its perpectivalism, namely its inbuilt dependence on measurement contexts. A review of this attempts is given and commented by Bitbol in [50]. Since RQM gives its full physical content to the aforementioned contextuality, through relationalism, it is easy to amend these arguments in direction of a relational explanation of quantum indeterminism.

As early as 1935, the philosopher Hermann made explicit this connection between context-dependence and indeterminism, by acknowledging the "relative character of the quantum-mechanical way of description" [51]. In this opuscule, she advocates the distinction between two distinct concepts, which we shall term 'predictive causality' and 'retrodictive causality'. She notices that, thanks to the von Neumann consistency of QM, retrodictive causality "is not only consistent with QM, but is demonstrably presupposed by it": When a specific outcome is obtained, it can always be traced back to a definite state of the system. (Notice the echo with RQM's claim that the von Neumann consistency is the core of QM.) What QM refuses to give us is only predictive causality. But this is no surprise, since the specification of the system's wave-function does not take into account the observing system relative to which the measured properties are defined. Illustrating Hermann's insight, we could get back once again to the Galilean analogy: How are we to interpret the lack of predictive causality of classical mechanics relative to questions such as "Given such preparation of the system, what is its velocity?"

The relation between perspectivalism and indeterminism has been even more clearly pointed out by Destouches-Février [52]. Indeed, she proved a theorem which states that any predictive theory bearing on phenomena defined relative to (incompatible) experimental contexts is "essentially indeterministic", and thus derives the Born rule. In the relational perspective, this means that the quantum indeterminacy stems from the necessity to relate physical properties to *distinct* observers.

5.2 The origin of collapse: self-referential censorship

The relational interpretation of QM provides a dualistic account of the measurement process: for an 'internal' observer O (by which we mean an observer actually interacting with the system S), the wave-function undergoes the usual 'collapse', implementing the actualization of a property. For an 'external' observer P (one which does not interact with S or O), on the contrary, the evolution of $S+O$ is purely unitary, Schödinger-like. This dualistic account of the same sequence of events has been acknowledged as the signature of the relational nature of quantum-mechanical properties. But this is not the whole story on this matter. RQM gives an explicit *explanation* for the breakdown of unitarity relative to O. This explanation is clearly identified in Rovelli's original paper [5], and deepened in the subsequent [6].

Unitarity and closedness

Where does the Schödinger equation come from? What is the physical meaning of unitarity? The answer should be obvious: unitarity is the translation of the time-translation invariance of *closed* systems. Now, relative to the internal observer O, the system S is precisely *not closed*: it interacts with O itself! Whence the breakdown of unitarity *relative to O*. To quote Rovelli [5]:

> "The unitary evolution does not breakdown for mysterious physical quantum jumps, or due to unknown effects, but simply because O is not giving a full dynamical description of the interaction. O cannot have have a full description of the interaction of S with himself (O), because his information is correlation, and there is no meaning in being correlated with oneself."

From the viewpoint of the external observer P, the situation is different. The compound system $S + O$ is closed, and thus evolves unitarily. This is all there is to the mystery of collapse.

Self-referential censorship

Actually, that is not all there is to it. One point deserves clarification: why can't O give a dynamical description of himself? This point has been addressed by Dalla Chiara [53] and Breuer [54]. Although space and time are lacking for a detailed account of these works, we reckon that a quotation from [6] briefly presenting them finds its place here:

> "M. Dalla Chiara has addressed the *logical* aspect of the measurement problem. She observes that the problem of self-measurement in QM is strictly related to the *self-reference* problem, which has an old tradition in logic. From a logical point of view the measurement problem of QM can be described as a characteristic question "semantic closure" of a theory. To what extent can QM apply consistently to the objects and the concepts in terms of which its metatheory is expressed? Dalla Chiara shows that the duality in the description of state evolution [...] can be given a purely logical interpretation: "If the apparatus observer O is an object of the theory, then O cannot realize the reduction of the wave-function. This is possible only to another O', which is 'external' with respect to the universe of the theory. In other words, any apparatus, as a particular physical system, can be an object of the theory. Nevertheless, *any apparatus which realizes the reduction of the wave-function is necessarily only a metatheoretical object.*" [53].
>
> [...] As is well-known, from a purely logical point of view self-reference properties in formal systems impose limitations on the descriptive power of the systems themselves. Breuer has shown that, from a physical point of view, this feature is expressed by the existence of limitations in the universal validity of physical theories, *no matter whether classical or quantum.* Breuer studies the possibility for an apparatus O to measure its own state. More precisely, of measuring the state of a system *containing* an apparatus O. He defines a map from the space of all sets of states of the apparatus to the space of all sets of states of the system. Such a map assigns to every set of apparatus states the set of system states that is compatible with the information that after the measurement interaction the apparatus is in one of these states. Under reasonable assumptions on this map, Breuer is able to prove a theorem stating that no such map can exist that can distinguish all the states of the system. An apparatus O cannot distinguish all the states of a system S containing O. This conclusion holds irrespective of the classical or quantum nature of the systems involved, but in the quantum context it implies that no quantum mechanical apparatus can measure all the quantum correlations between *itself* and an external system. These correlations are only measurable by a second external apparatus, observing both the system and the first apparatus."

Chapter 6

The 'EPR paradox' dissolved

In this chapter[1], we argue that in the context of Rovelli's interpretation of QM, it is not necessary to abandon locality in order to account for EPR correlations. From the relational perspective, the apparent "quantum non-locality" is a mistaken illusion caused by the error of disregarding the quantum nature of *all* physical systems.

The price for saving locality is the weakening of realism which is the core of RQM. In fact, the historical development of the EPR debate testifies to a significant mutation of the stakes of the EPR argument. In the original 1935 EPR article [13], this argument was conceived as an attack against the description of measurements in Copenhagen quantum theory, and a criticism to the idea that Copenhagen QM could be a *complete* description of reality. Completeness, locality, and a strong form of realism were argued by EPR to be incompatible with QM predictions. With Bell's contribution [14], the argument has been mostly reinterpreted as a challenge to "local realism". More recently, especially after the Kochen-Specker theorem [15], it is the very possibility of uncritically ascribing "properties" to a quantum system, that has been questioned. The problem of locality has thus moved to the background, replaced by a mounting critique of strong notions of reality (see for instance [16]).

Here we take this conceptual evolution to what appears to us to be its necessary arriving point: the possibility that EPR-type experiments disprove Einstein's strong realism, rather than locality[2].

6.1 Locality and separability in relational quantum mechanics

Locality

Recall that *locality* is the principle demanding that *two spatially separated objects cannot have instantaneous mutual influence*. We will argue that this is not contradicted by EPR-type correlations, if we take the relational perspective on quantum mechanics.

In fact, locality is at the roots of the observation that different observers do not describe the same reality. As emphasized by Einstein, it is locality that makes possible the individuation of physical systems, including those we call observers[3] .

Distinct observers that are distant and not interacting do not describe the same reality. Even beyond its foundation role in relativistic field theories, locality constitutes, therefore, the base of the relational methodology: an observer must not, and cannot, account for events involving systems located out of its causal neighborhood (or light-cone).[4]

Separability

Another concept playing an important role in the EPR discussions is *separability*. An option that saves a (weakened) form of locality is, according to some, to assume that entangled quantum objects are "not-separable". Aspect, for instance, writes that "a pair of twin entangled photons must in fact be regarded as a single, *inseparable* system, described by a global quantum state" [29]. This rather strange conception, where two

[1]The content of this chapter is a superficially edited version of a paper I cosigned with Rovelli, entitled *Relational EPR*. It is available on the arXiv: quant-ph/0604064, and has been submitted for publication to *Foundations of Physics Letters*. Therefore, arguments and examples presented in this chapter should not be attributed to myself only, but to the both of us.

[2]Similar views have been recently expressed by a number of authors [17, 18, 19, 20, 21].

[3]"Without the assumption of the mutually independent existence (the 'being-thus') of spatially distant things, an assumption which originates in everyday thought, physical thought in the sense familiar to us would not be possible. Nor one does can see how physical laws could be formulated and tested without such a clean separation." Quoted in [28], where this point is discussed in depth.

[4]We can take this observation as an echo in fundamental physics of the celebrated: "7. Whereof one cannot speak, thereof one must be silent" [24].

systems are actually a single system, indicates, in our opinion, the difficulty to conciliate realism, locality and quantum theory.

We will argue below that the abandonment of Einstein's strict realism allows one to exempt himself from this type of intellectual acrobatics.

Let us instead formulate the following definition of separability:

Definition 3 *(Separability) Two physical systems S_1 and S_2 are* separable *if there exist a complete set of observables (in the sense of Dirac) of the compound system S_1+S_2 whose values can be actualized by measurements on S_1 or S_2 only. Such observables are called* individual observables*; the others are called* collective observables*.*

This notion of separability is equivalent to a minimal operational definition of subsystems of a composite system. It is deliberately weak (and in the end trivial); any stronger definition testifies to some unease towards reality.

6.2 Relational discussion of the EPR argument

Bohm's setting of the experiment

Consider a radioactive decay, producing two spin-half particles, and call them α and β. Suppose that some previous measurement ensures that the square of the total spin of the two particles equals to zero —which corresponds, in the spectroscopic vocabulary, to the singlet state. The particles α and β leave the source in two different directions, reaching two distant detectors A and B, which measure their spin in given directions.

Einstein's counterfactual argument

According to QM, the measurement of an observable provokes the projection of the system's state onto the eigenspace associated with the eigenvalue obtained. In the case of the singlet, the state can be equivalently decomposed on the eigenbasis of the spin in two different directions, say z and x:

$$\begin{aligned} \left|\psi_{\text{singlet}}\right\rangle &= \frac{1}{\sqrt{2}}\left(\left|\downarrow\right\rangle_\alpha \left|\uparrow\right\rangle_\beta - \left|\uparrow\right\rangle_\alpha \left|\downarrow\right\rangle_\beta\right) \\ &= \frac{1}{\sqrt{2}}\left(\left|\rightarrow\right\rangle_\alpha \left|\leftarrow\right\rangle_\beta - \left|\leftarrow\right\rangle_\alpha \left|\rightarrow\right\rangle_\beta\right). \end{aligned} \tag{6.1}$$

Depending on whether the observer A measures the spin of α in the direction z or x, the second particle β finds itself in an eigenstate of S^z or S^x. In either case, the property of having a definite spin in one direction is uniquely determined for β, hence is real, since, according to Einstein's realism,

> *If, without in any way disturbing a system, we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity. [13]*

But according to the principle of locality, the choice made by A cannot have an influence on β, which is space-like separated from A. Therefore, in order to accommodate both possibilities it is necessary for the spin in *both* directions to be uniquely determined. But this is more physical information than the one contained in a vector in the Hilbert space of the states of β. Hence there exist real properties not described by quantum mechanics. Completeness of quantum mechanics, namely one-to-one correspondence between the mathematical objects used to describe the state of a system and its real state, is disproved.

From incompleteness to non-locality

EPR-type experiments, championed by Aspect *et al.* [1, 2], are often interpreted as empirical evidence for the existence of a somewhat mysterious "quantum non-locality". For instance, Chris Isham concludes his exposition of the EPR debate with the words "[...] we are obliged either to stick to a pragmatic approach or strict instrumentalist interpretation, or else to accept the existence of a strange non-locality that seems hard to reconcile with our normal concepts of spatial separation between independent entities" [3]. On a similar vein, Travis Norsen writes recently: "What Bell's theorem proves is that the predictions of quantum mechanics for certain experimental results —predictions that have stood up to the test of experiment— are inconsistent with the principle of local causality, period" [4]. In spite of seven decades of reflection on this problem, the precise nature of this non-locality remains rather elusive.

Critique of Einstein's argument

Einstein's argument relies on the strongly realistic hypothesis that *the actual properties of the particles (the "real state of affairs") revealed by the detectors are observer-independent.* It is this hypothesis that justifies the ascription of a definite, objective, state to each particle, at every instant of the experiment: in Einstein's account, when B measures the spin of β, the measured value instantaneously acquires an objective existence for A. This hypothesis is not physically justified: nothing enables A to know the outcome of the measure carried out by B on β, unless A measures the state of B. But A cannot measure the state of B instantaneously, *precisely* because of locality: B is far away.

What is missing in Einstein's quotation above is the distinction between "elements of physical reality" (quantum events) relative to A and "elements of physical reality" relative to B.

Observer A can of course measure the state of B (or, for that matter, β), but only when A is back into causal contact with B. This is, needless to say, in the future light-cone of A, and therefore poses no challenge for locality. In other words, Einstein's reasoning requires the existence of a hypothetical super-observer that can instantaneously measure the state of A and B. It is the hypothetical existence of such nonlocal super-being, and not QM, that violates locality[5].

Let us look at the origin of the mistake more in detail. Suppose that A measures a spin component of α at time t_0, and B measures a spin component of β at time t'_0. Einstein's ingenious counterfactual argument works under the assumption that locality prevents any causal influence of A's measurement on B's (A's choice of measuring the spin along z or along x cannot affect the B measurement, hence we can counterfactually join the consequences of the two alternatives). But for such counterfactuality to be effective, there has to exist an objective element of reality which is unaffected by A's actions. Indeed, if one acknowledges that B's state of affairs is *a priori* undefined for A, then bringing B into the argument is useless, because then what would be actualized by A's measurement of the spin of α along one direction would be *relative to A* only. In fact, Einstein implicitly assumes that B is a *classical* system, recording *objective* values in its "pointer variables". In this case, the properties of β become actual when it interacts with B at time t'_0, indeed substantiating the non-local EPR correlations between distant locations. Thus, *it is the assumption that B is classical and fails to obey quantum theory that creates EPR non-locality.*

But all systems are quantum: there are no intrinsically classical systems. Hence the hypothesis that B does not obey quantum theory is not physically correct. It is this mistaken hypothesis that causes the apparent violation of locality.

In other words, in the sequence of events which is real for A there is no definite quantum event regarding β at time t_0, and therefore no element of reality generated non-locally at time t_0 in the location where B is. Hence Einstein's argument cannot even begin to be formulated.

What changes instantaneously at time t_0, for A, is not the objective state of β, but only its (subjective) relative state, that codes the information that A has about β. This change is unproblematic, for the same reason for which my information about China changes discontinuously anytime I read an article about China in the newspaper. Relative to A, β is not affected by this change because there is no β-event happening at time t_0. The meaning of the sudden change in the state of β is that, as a consequence of her measurement on α, A can predict outcomes of *future* measurement that A herself might do on β, or on B.

Of course the price to pay for this solution of the puzzle is that the sequence of events which is real for B is different from the one which is real for A. For B, there *is* a quantum event of β at time t'_0 and there is *no* quantum event regarding α at time t'_0.

Relational discussion of the EPR experiment

We shall now present a relational discussion of the EPR experiment, compatible with locality. But first, let us get rid of the problem of separability: in the EPR experiment, the two entangled systems interact with two different observers. Incontestably, both get definite outcomes during these complete measurements (in the sense of Dirac). Hence, the particles are separable. Fine - one might say - but what about the EPR correlations?

Individual measurements

Say that A measures the spin of α in the direction n at time t_0. This is an individual observable, denoted $S^n_{A\alpha}$. Suppose B measures the spin of β in the direction n' at time $t^{'}_0$ (individual observable $S^{n'}_{B\beta}$). Let us denote $\epsilon_{A\alpha}$ and $\epsilon_{B\beta}$ ($\epsilon = \pm 1$) the corresponding outcomes. Because A and B are space-like separated, there cannot exist an observer with respect to which both of these outcomes are actual, and therefore it is meaningless to compare $\epsilon_{A\alpha}$ and $\epsilon_{B\beta}$: A's outcome is fully independent of B's, and *vice versa.*

[5]A similar implicit hypothesis, the "retrodiction principle", is pointed out by Bitbol in [17].

EPR correlations

But these individual measures do not exhaust all possibilities. In the EPR experiment, the composite system $\alpha+\beta$ is assumed to be in the singlet state. From the relational point of view, this means that some observer, say A, has the information that the total spin of $\alpha+\beta$ equals to zero. That is, it has interacted with the composite system in the past and has measured the square of the total spin. Let us call this *collective* observable $S^2_{A,\alpha+\beta}$.

The measurement of $S^n_{A\alpha}$ brings new information to A. It determines the change of the relative state of α. Notice that A's knowledge about α changes (epistemic aspect), and, at the same time, A' predictions concerning future change (predictive aspect). For instance, A becomes able to predict with certainty the value of $S^n_{A\alpha}$ if the interaction is repeated.

But there is another observable whose value QM enables A to predict: $S^{n'}_{A\beta}$, namely the measurement that A can perform on β at the time t_1, when β is back into causal contact with A. For instance, if

$$S^2_{A,\alpha+\beta} = 0 \quad \text{and} \quad S^n_{A\alpha} = \epsilon, \tag{6.2}$$

then QM predicts

$$S^n_{A\beta} = -\epsilon. \tag{6.3}$$

That is, knowledge of the value of the collective observable $S^2_{\alpha+\beta}$ plus knowledge of the individual observable $S^n_{A\alpha}$ permits to predict the future outcome of the individual observable $S^n_{A\beta}$: it is *this* type of inference which constitutes the "EPR correlations". It concerns a sequence of causally connected interactions.

Consistency

Let us bring B back into the picture. It is far from the spirit of RQM to assume that each observer has a "solipsistic" picture of reality, disconnected from the picture of all the other observers. In fact, the very reason we can do science is because of the consistency we find in nature: if I see an elephant and I ask you what you see, I expect you to tell me that you too you see an elephant. If not, something is wrong.

But, as claimed above, any such conversation about elephants is ultimately an interaction between quantum systems. This fact may be irrelevant in everyday life, but disregarding it may give rises to subtle confusions, such as the one leading to the conclusion of nonlocal EPR influences.

In the EPR situation, A and B can be considered two distinct observers, both making measurements on α and β. The comparison of the results of their measurements, we have argued, cannot be instantaneous, that is, it requires A and B to be in causal contact. More importantly, with respect to A, B is to be considered as a normal quantum system (and, of course, with respect to B, A is a normal quantum system). So, what happens if A and B compare notes? Have they seen the same elephant?

It is one of the most remarkable features of quantum mechanics that indeed it automatically guarantees precisely the kind of consistency that we see in nature [5]. Let us illustrate this assuming that both A and B measure the spin in the same direction, say z, that is $n = n' = z$.

Since B is a quantum system, there will be an observable S^n_{AB} corresponding to B's answer (at time t_1) to the question "which value of the spin have you measured?". That is, S^n_{AB} is the observable describing the pointer variable in the detector B. Then consistency demands that:

(i) If A measures S^n_{AB} after having measured $S^n_{A\beta}$, she will get

$$S^n_{AB} = S^n_{A\beta}. \tag{6.4}$$

(ii) If a third observer C, who has the prior information that measurements have been performed by A and B, measures at a later time the two pointer variables: S^n_{CA} and S^n_{CB} then

$$S^n_{CB} = -S^n_{CA}. \tag{6.5}$$

But this follows from standard QM formalism, because an interaction between β and B that can be interpreted as a measurement is an interaction such that the state (1) and the initial state of α, β and B evolve into the state (relative to A)

$$|\psi\rangle^{(A)}_{\alpha+\beta+B} = \frac{1}{\sqrt{2}}\Big(|\downarrow\rangle_\alpha |\uparrow\rangle_\beta |\uparrow\rangle_B - |\uparrow\rangle_\alpha |\downarrow\rangle_\beta |\downarrow\rangle_B\Big) \tag{6.6}$$

with obvious notation. Tracing out the state of α that plays no role here, we get the density matrix

$$\rho^{(A)}_{\beta+B} = \frac{1}{2}\Big(|\uparrow\rangle_\beta |\uparrow\rangle_B \langle\uparrow|_\beta \langle\uparrow|_B + |\downarrow\rangle_\beta |\downarrow\rangle_B \langle\downarrow|_\beta \langle\downarrow|_B\Big). \tag{6.7}$$

from which (4) follows immediately. Similarly, the state of the ensemble of the four systems α, β, A, B, relative to C evolves, after the two interactions at time t_0 into the state

$$|\psi\rangle^{(C)}_{\alpha+\beta+A+B} = \frac{1}{\sqrt{2}}\Big(|\downarrow\rangle_\alpha |\uparrow\rangle_\beta |\downarrow\rangle_A |\uparrow\rangle_B - |\uparrow\rangle_\alpha |\downarrow\rangle_\beta |\uparrow\rangle_A |\downarrow\rangle_B\Big) \tag{6.8}$$

again with obvious notation. Tracing out the state of α and β, we get the density matrix

$$\rho^{(C)}_{A+B} = \frac{1}{2}\Big(|{\downarrow}\rangle_A |{\uparrow}\rangle_B \langle{\downarrow}|_A \langle{\uparrow}|_B + |{\downarrow}\rangle_A |{\uparrow}\rangle_B \langle{\downarrow}|_A \langle{\uparrow}|_B \Big). \tag{6.9}$$

which gives (5) immediately. It is clear that everybody sees the same elephant. More precisely: everybody hears everybody else stating that they see the same elephant he sees. This, after all, is the best definition of objectivity.

An objection

An instinctive objection to the RQM account of the above situation is the following[6]. Suppose that at a certain time the following happens

> (*) *A observes the spin in a given direction to be $\uparrow$ AND B observes the spin in the same direction to be also $\uparrow$.*

Agreement with quantum theory demands that when later interacting with B, A will necessarily finds B's pointer variable indicating that the measured spin was $\downarrow$. This implies that what A measures about B's information ($\downarrow$) is unrelated to what B has actually measured ($\uparrow$). The conclusion appears to be that each observer sees a completely different world, unrelated to what any-another observer see: A sees an elephant and hears B telling her about an elephant, even if B has seen a zebra. Can this happen in the conceptual framework of RQM?

The answer is no. The reason is subtle and it is at the core of RQM.

The founding postulate of RQM is to stipulate that we shall not talk about properties of systems in the abstract, but only of properties of systems relative to *one* system. In particular, we can never juxtapose properties relative to different systems. If we do so, we make the same mistake as when we simultaneously ascribe position and momentum to a particle. In other words, RQM is *not* the claim that reality is described by the *collection* of all properties relatives to all systems. This collection is assumed not to make sense. Rather, reality admits one description per each (observing) system, each such description is internally consistent.

In turn, each given system can be observed by another system. RQM is, in a sense, the stipulation that we shall not talk about anything else than that, and the observation that this scheme is perfectly sufficient for describing nature and our own possibility of exchanging information about nature (hence circumventing solipsism).

So, the case (*) can never happen, because it does not happen neither with respect to A nor with respect to B. The two sequences of events (the one with respect to A and the one with respect to B) are distinct accounts of the same reality that cannot and should not be juxtaposed. The weakening of realism is the abandonment of the unique account of a sequence of the events, and its replacement with compatible alternatives, not with a self-consistent collection of all relative properties.

Once more, this does not mean that B and A cannot communicate their experience. In fact, in either account the possibility of communicating experiences exists and in either account consistency is ensured. Contradiction emerges only if, against the main stipulation of RQM, we insist to believe that there is an absolute, external account of the state of affairs in the world, obtained by juxtaposing realities relative to different observers.

Comparison with Laudisa's discussion of relational EPR

The EPR argument has been discussed in the context of relational quantum mechanics also by Federico Laudisa, in a recent paper [30]. Laudisa's discussion has some points in common with the one given here, but it differs from the present one in one key respect.

Laudisa starts with a reformulation of the EPR hypotheses, namely realism, locality and completeness of QM, in a form meant to be compatible with RQM. The locality principle, in particular, is given the following formulation: *No property of a physical system S that is objective relative to some observer can be influenced by measurements performed in space-like separated regions on a different physical system.* He is then able to show that the contradiction between locality, formulated in this manner, and QM is *itself* relative, in the sense that it is frame-dependent: there is always an observer (in the sense of special relativity) for which it is inexistent.

Laudisa's argument

Here goes Laudisa's argument: after the measurement of the spin of α (say in the direction z) by A, the spin of β (in the same direction z) has a determined valued relative to A. However, according the (relativized) locality principle, β cannot acquire a property relative to A as a consequence of the measure performed on α. Hence, relative to A, the spin of β already had a determined value *before* the measurement. This fact is in contradiction

[6] This paragraph was written by Rovelli, and not myself. We add it here because of its pedagogical value.

with the prior state of the compound system relative to A, $|\psi\rangle^{(A)}_{\alpha+\beta} = \frac{1}{\sqrt{2}}(|\downarrow\rangle_\alpha |\uparrow\rangle_\beta - |\uparrow\rangle_\alpha |\downarrow\rangle_\beta)$, which leads to the improper mixture representing the state of β relative to A: $\rho^{(A)}_\beta = \frac{1}{2}(|\uparrow\rangle_\beta \langle\uparrow|_\beta + |\downarrow\rangle_\beta \langle\downarrow|_\beta)$. At that point, Laudisa remarks that, because of space-like separation between A and B, one can find a reference frame in which A's measurement precedes B's. In such a frame, when A faces the locality/completeness contradiction, B has not performed any measurement yet, and therefore escapes the contradiction. What is more, there exists another reference frame in which the chronology of measurements is inverted, so that the contradiction afflicts B but not A. Finally, the EPR contradiction turns out to be frame-dependent, and thus fails to refute the locality principle in an absolute sense.

Comparison

Laudisa's interpretation is based on the same premise — relativity of quantum states —, but differs from the one presented here. Unlike Laudisa, we do not understand locality as prohibiting the acquisition of information by an observer on a distant system, but only as prohibiting the possibility that a measurement performed in a region could, in any way, affect the outcomes of a measurement happening in a distant region. In the EPR scenario we have discussed, the state of β relative to B is independent of A's measurement, but not the state of β relative to A. Since the existence of correlations between α and β is know *a priori* by A, the measurement of an individual observable of α does permit the prediction of the value of an individual observable of β. What is affected by this measurement is not a hypothetical absolute physical state of β, but just A's knowledge about β. It is B's knowledge (or direct experience) about β that cannot be affected by anything performed by A.

Laudisa's residual frame-dependent contradiction between locality and completeness results from an interpretation of locality which disregards the epistemic aspect of relative states. More radical, our conclusion here is that there is no contradiction at all between locality and completeness, nor, more in general, locality and QM predictions.

Part III

On the road to “relational quantum logic”

Chapter 7

Deconstructing Hilbert spaces: quantum logic

In an operationalist approach to science, it is of major importance to formulate definitions in terms which make explicit their connection to physical processes. The notion which plays this semantic role in quantum physics is that of projection operators. von Neumann was the first to emphasize their fundamental importance, initiating the stream of research called 'quantum logic'. The latter can be defined as the attempt to point out the origin of quantumness at the most primitive level, that of experimental propositions of the type "the system possesses such property". This is done by shifting the attention to the algebraic structure of the set of projectors in the Hilbert space.

A tentative definition of operational quantum logic is given in [47]:
"Operational quantum logic involves

1. the fact that the structure of the 2-valued observables in orthodox QM may usefully be regarded as a non-classical propositional logic,
2. the attempt to give independent motivation for this structure, as part of a general program to understand quantum mechanics, and
3. the branch of pure mathematics that has grown out of (1) and (2), and now concerns itself with a variety of "orthomodular" structures generalizing the logic of 2-valued quantum observables."

In this chapter, we shall present the basic concepts of operational quantum logic, without explicitly relating them to the relational interpretation of QM. This step will be taken in the next chapter, where we will use them to investigate the possibility of a relational reconstruction of QM. As we shall see, this relational reappraisal of quantum logic will require a profound modification of its interpretation.

7.1 From quantum states to experimental propositions

Inadequacy of Hilbert space representation

As well-known, the axiomatization of QM in terms of Hilbert spaces was achieved by von Neumann in the late 20's (von Neumann 1932 book). This great achievement impressively enhanced the predictive power of QM, so much so that the interpretational issues of the early years rapidly fell in abeyance.

Despite its uncountable successes, the Hilbert space representation of QM does suffer from a number of serious weaknesses. Noteworthily, it was von Neumann himself who first realized this fact. Here are his words [31]:

> "I would like to make a confession which may seem immoral: I do not believe absolutely in Hilbert space any more. [...] Now we begin to believe that it is not the *vectors* which matter, but the lattice of all linear (closed) subspaces. Because: 1) The vectors ought to represent the physical *states,* but they do it redundantly, up to a complex factor, only, 2) and besides, the states are merely a derived notion, the primitive (phenomenologically given) notion being the qualities which correspond to the *linear closed subspaces."*

This remark expresses the spirit of quantum-logical approaches: Hilbert spaces are not the appropriate framework to study the operational foundation of the quantum formalism. In order to shed some light on the mathematics of QM, one must find, and analyze, more fundamental structures than Hilbert spaces.

Experimental propositions

This "deconstructivist" strategy was championed by Birkhoff and von Neumann in a seminal paper of 1936 [32]. Their key idea is to trace back specifically quantum behaviors, such as non-commutativity of observables, to the very logical structure of the theory. Focusing on primitive notions such as *experimental propositions, or questions*, one can expect to dig out a general framework in which both classical and quantum mechanics could be inserted — and compared.

Experimental propositions are the simplest observables of a system, for they have exactly two possible outcomes. They correspond to propositions of the form "$A \in \Delta$", where A is an observable and $\Delta \subset \mathbb{R}$. Their operational definition is obvious: any physical experiment amounts to answering (at least) one such question. Logical operations between questions (negation, conjunction, disjunction) induce an algebraic structure structure on the set $\mathcal{L}$ of all questions. It is this algebraic structure which Birkhoff and von Neumann propose to investigate in both the classical and the quantum case.

Questions in classical mechanics

In classical mechanics, the fundamental structure representing the accessible states ϕ is the phase space Γ. The observables are just the (Borel) functions defined on this phase space. Now, given such an observable $A : \Gamma \to \mathbb{R}$, an experimental proposition $q =$ '$A \in \Delta'$ is represented by the (Borel) subset of Γ defined by: $\gamma_q = A^{-1}(\Delta)$. (Indeed, the proposition q is true in the state ϕ if and only if $A(\phi) \in \Delta$.)

Hence, in the classical case, $\mathcal{L}$ is just the set of all (Borel) subsets of Γ: $\mathcal{L} = \mathcal{B}(\Gamma)$. It is straightforward to see that the logical connectives, negation, conjunction and disjunction, are represented respectively by set-theoretic complement, intersection and union: $\gamma_{\neg q} = \gamma_q^c$, $\gamma_{p\wedge q} = \gamma_p \cap \gamma_q$, $\gamma_{p\vee q} = \gamma_p \cup \gamma_q$. Thus, the algebraic structure of the classical model for the propositional calculus of experimental propositions is that of a *Boolean algebra*, namely of the Boolean algebra $(\mathcal{L}, ^c, \cap, \cup)$[1].

Questions in quantum mechanics

von Neumann and Birkhoff's celebrated contribution to the foundations of QM was, among other things, to identify the analogue of the Boolean algebra of questions $(\mathcal{L}, ^c, \cap, \cup)$ in the quantum formalism. Here, experimental propositions appear to be represented by self-adjoint projection operators in the Hibert space $\mathcal{H}$. Indeed, recall that a question is a two-valued observable, and that a self-adjoint operator with exactly two elements in its spectrum is a projector. Given an observable A, the projector associated to the question $q = A \in \Delta$ is given by the spectral element: $\gamma_q = \chi_\Delta(A) = \sum_{a\in\Delta\cap Sp(A)} |a\rangle\langle a|$ (where χ_Δ denotes the characteristic function of Δ). According to the statistical algorithm of QM, the experimental proposition $q = A \in \Delta$ is true in state ψ iff $\gamma_q\psi = \psi$ (eigenvalue '1'), and false iff $\gamma_q\psi = 0$ (eigenvalue '0').

Thus, in QM, the set of questions $\mathcal{L}$ is the set $\mathcal{P}(\mathcal{H})$ of self-adjoint projectors in $\mathcal{H}$. Since there is a 1-to-1 correspondence between self-adjoint projectors and (closed) subspaces of $\mathcal{H}$ (associating to each projector γ_q its range $\bar{\gamma}_q$), $\mathcal{L}$ can be equivalently taken bo be the set $\mathcal{C}(\mathcal{H})$ of (closed) subspaces of $\mathcal{H}$: $\mathcal{L} = \mathcal{C}(\mathcal{H})$. Now, how are represented the logical connectives in this case?

We have just seen that the experimental proposition $q = A \in \Delta$ is false in state ψ iff $\gamma_q\psi = 0$. This is clearly equivalent to $\psi \in \bar{\gamma}_q{}^\perp$, so that $\neg q$, being true iff q is false, is represented by $\bar{\gamma}_q{}^\perp$: $\gamma_{\neg q} = \bar{\gamma}_q{}^\perp$. Now for the conjunction: given a state ψ, p and q are simultaneously true iff $\psi \in \bar{\gamma}_p$ and $\psi \in \bar{\gamma}_q$, i.e. $\psi \in \bar{\gamma}_p \cap \bar{\gamma}_q$. Hence: $\bar{\gamma}_{p\wedge q} = \bar{\gamma}_p \cap \bar{\gamma}_q$. The same reasoning applies to the disjunction: p or q is true iff $\psi \in \bar{\gamma}_p \cup \bar{\gamma}_q$. We need to be careful here, because $\bar{\gamma}_p \cup \bar{\gamma}_q$ is not a (closed) subspace of $\mathcal{H}$. Since the smallest such (closed) subspace containing $\bar{\gamma}_p \cup \bar{\gamma}_q$ is the (closed) linear sum of $\bar{\gamma}_p$ and $\bar{\gamma}_q$, denoted $\bar{\gamma}_p \bar{+} \bar{\gamma}_q$, one actually has: $\bar{\gamma}_{p\vee q} = \bar{\gamma}_p \bar{+} \bar{\gamma}_q$.

Finally, the logically structured set of quantum mechanical questions is $(\mathcal{C}(\mathcal{H}), ^\perp, \cap, \bar{+})$. Its algebraic structure is not that of a Boolean algebra. Because it does not resemble anything previously know, one terms it laconically a *Hilbert lattice*[2].

7.2 Peculiarity of the quantum logic

The purpose of the previous analysis was to dig out the underlying logical structure of QM. The point of this work, von Neumann and Birkhoff's motivation, is to compare at the deepest level the differences between classical mechanics and QM. And, hopefully, to understand at last the peculiarities of quantumness. The difference they identified is the non-distributivity of the quantum propositional calculus.

[1] A Boolean algebra is a set B supplied by an involution $\neg$ and two binary operations on B, satisfying axioms modeling the propositional calculus of classical logic. Among them, one (the distributivity axiom) is essential: for $a, b, c \in B$, $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$. By the definition, the Borel subsets of a set form such a Boolean algebra

[2] The word 'lattice' is a generic word for algebraic structures where two binary operations (mimicking conjunction and disjunction) are defined.

Non-distributivity

Distributivity is an essential property of classical logic (and, actually, part of the definition of a Boolean algebra). It states that, for any three propositions p, q, r:

$$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r) \tag{7.1}$$

For the quantum structure $(\mathcal{C}(\mathcal{H}),^{\perp}, \cap, \bar{+})$, this property fails to hold. Instead, one has the weaker property of 'orthomodularity', which states that for any two propositions p, q such that $p \wedge q = p$[3]:

$$p \vee (\neg p \wedge q) = q \tag{7.2}$$

Thus, the lattice of quantum questions is globally non-distributive. Nevertheless, distributivity holds in special cases, *e. g.* when the questions p and q are such that the subspaces γ_p and γ_q are orthogonal. That is to say that $\mathcal{L}$ is partially (or 'locally') Boolean. The logic of QM is locally classical, but globally non-classical.

Consequence and interpretation

von Neumann and Birkhoff's algebraic dissection of $\mathcal{C}(\mathcal{H})$ thus reveals that a non-distributive lattice plays a fundamental role in the logical structure of the theory. Since this structure differs from the Boolean algebra of classical semantics, the question arises: what is the locus of this difference? Two answers have been given to this question.

The first one, which is attributed to Putnam and Finkelstein, proposes that the discovery of quantum logic invalidates the applicability of classical reasoning for describing the world. The situation would be analogue to the discovery of the non-Euclidian character of space geometry by Einstein: Boolean logic, like Euclidian geometry, is empirically inadequate, and must be thought of as a mere approximation of the true, quantum logic. Therefore, the 'laws of thought' are a matter of empirical science, submitted to revision, and not some kind of transcendental structure of the mind.

According to this interpretation some usual inferences which rely on the distributive law turn out to be invalid. For instance, "p or q, not q, therefore q" is just *not* a valid inference [48]. However, both *modus ponens* ("if p then q, p, therefore q") and *modus tolens* ("if p then q, not q, therefore not p") remain valid forms of argument.

The other interpretation of quantum logic, tracing back to Mackey, suggests that the non-distributivity of quantum logic does not threaten the classical laws of thought, but rather reflects the fact that not all phenomena can be observed simultaneously. Thus, in this operational interpretation of quantum logic, the non-Boolean structure of the quantum $\mathcal{L}$ mirrors the structure of quantum experiments themselves: one is confronted with a non-classical 'logic of quantum operations'.

7.3 Mackey's reconstruction program

Characterizing Hilbert lattices

The interest of pointing out algebraic properties of Hilbert lattices, such as orthomodularity, goes beyond the sole analysis of the quantum formalism and its underlying logic as they stand. Indeed, Mackey, in a seminal paper, argued that the complete algebraic characterization of Hilbert lattices should be a goal of quantum logic *per se*. Why so?

Mackey was one of the first to emphasize that QM will cease to be puzzling only when we are able to reconstruct it from operationally well-defined first principles. This said, the question arises: what need we reconstruct? Mackey answers is 'a Hilbert lattice', *i.e.* a lattice of questions which is isomorphic to the set of all (closed) subspaces of a Hilbert space. Indeed, once we have on hand such a Hilbert lattice, it is straightforward to reconstruct the whole quantum formalism. In the next paragraph, we shall give a sketch of this reconstruction. But before, let us summarize Mackey's program:

1. find a complete algebraic characterization of Hilbert lattices, *i.e.* find a set of axioms for the set of questions of a system entailing its isomorphism with the set of (closed) subspaces of a Hilbert space
2. give an operational meaning for these axioms

After more than 30 years of research, point 1 has been achieved[4]. Unfortunately, it is not the case of point 2. Some of the algebraic axioms characterizing Hilbert lattices remain physically unclear. It will be the subject of the next chapter to see how RQM sheds a new light on the reconstruction program. But before, let us explain in what sense a Hilbert lattice is all we need to reconstruct QM.

[3]To prove this, one has to show that given two (closed) subspaces of a Hilbert space A, B, with $A \subset B$, satisfy $A\bar{+}(A^{\perp} \cap B) = B$, which is trivial.

[4]The characterization in question, worked out by Piron and Soler, involves abstract mathematics, which prevent us from sensibly presenting it here. We refer the reader to [7] for more on this topic.

Reconstructing the quantum formalism

As explained in [47], Mackey's essential remark is that both observables and states of QM can be easily constructed from a Hilbert lattice $\mathcal{L} \simeq \mathcal{C}(\mathcal{H})$, as a sort of non-Kolmogorovian probability calculus. Dynamics then follow from symmetry principles.

Reconstructing states

First, recall the statistical algorithm of QM: a (possibly impure) state ρ acts as a probability measure on $\mathcal{L}$, explicitly given by the Born rule: $\mathcal{L} \ni P \mapsto Tr(\rho P)$[5]. Now, Mackey argues, we can *define* a state to be a probability measure ω on $\mathcal{L}$. Gleason's theorem [45] then entails that there exists a density matrix ρ such that ω has the Born form $\omega(.) = Tr(.\rho)$. In this way we recover the state space of QM (density matrices on $\mathcal{H}$).

Reconstructing observables

Second, recall the spectral theorem: any observable A can be represented as an application $\alpha_A : Sp(A) \mapsto \mathcal{L}$, associating to each eigenvalue a_i the projector P_{a_i} on the corresponding eigenspace of $\mathcal{H}$. The application α_A is such that: (i) $\alpha_A(Sp(A)) = Id$ (ii) $\alpha_A(\cup_i \Delta_i) = \sum_i \alpha_A(\Delta_i)$ when the Δ_i's are disjoint (Borel) subsets of $Sp(A)$. In other words, the observable A can be represented by an $\mathcal{L}$-valued probability measure on $Sp(A)$. Thus, following Mackey, we can *define* an observable α_A to be an $\mathcal{L}$-valued probability measure on $\mathbb{R}$. To reconstruct the usual self-adjoint operator A, we simply write: $A = \int_{\mathbb{R}} s d\alpha_A(s)$.

Reconstructing unitary dynamics

Time translation is a symmetry for a closed quantum system (it is probability-preserving). Focusing on the dynamics or pure states, Wigner's theorem on representations of symmetry groups provides us with a one-parameter group $\{U_t\}$ of unitary/anti-unitary operators acting on $\mathcal{H}$. The anti-unitary case can be excluded be topological arguments (anti-unitary operators are not connected to the identity). Hence, we have a (assumedly continuous) one-parameter group of unitary operators $\{U_t\}$. Stone's theorem then allows us to write U_t as e^{-iHt}, where H is a self-adjoint operator – the Hamiltonian. This is precisely the (integrated) Schrödinger equation.

Thus, as announced, a Hilbert lattice is sufficient to reconstruct most of the QM formalism (including the Born rule). This is why many attempts have been made to get this Hilbert lattice from first principles. None of them has been completely successful The next chapter investigates the role RQM could play in this direction.

[5] In this context, a probability measure is defined as a nonnegative real function ω on $\mathcal{L}$, such that: (i) $\omega(\mathcal{H}) = 1$ and (ii) whenever A_i are pairwise orthogonal elements of $\mathcal{L}$, $\omega(\cup_i A_i) = \sum_i \omega(A_i)$.

Chapter 8

Some steps towards a relational quantum logic

In the previous chapter, we have presented a non-classical propositional calculus, constructed on the set $\mathcal{C}(\mathcal{H})$ of (closed) subspaces of a Hilbert space. We have also introduced the standard semantics associated to this propositional calculus, which asserts the correspondence between the elements of this language (the subspaces of $\mathcal{H}$) and experimental propositions of the form '$A \in \Delta$'. The analysis of the mathematical structure $\mathcal{C}(\mathcal{H})$ left us amazed by the non-distributivity of the quantum 'or' with respect to the quantum 'and'.

Inspired by the work of Heelan [48], we argue in this chapter that this view, which assumes that the essence of quantumness lies in the non-distributivity of experimental questions, is misleaded, and propose a different physical interpretation of the quantum propositional calculus. The purpose of this analysis is of course to address the task of reconstructing QM from relational premisses. Notice that such an attempt is made by Rovelli in his original paper [5]. However, the approach envisaged here departs from Rovelli's sketch[1]. For an account of Rovelli's reconstruction, we refer the reader to the aforementioned reference, and to [8].

8.1 Criticisms to standard operational quantum logic

The standard view of quantum logic is based on the identification of a lattice of questions $\mathcal{C}(\mathcal{H})$ in the Hilbertian formalism. The lattice structure of $\mathcal{C}(\mathcal{H})$ is defined by the operations $\cap$ and $\bar{+}$, which are thought of as linear-algebraic models of the logical conjunction $\wedge$ and disjunction $\vee$. One difficulty with this approach is the lack of a clear operational definition of these connectives; another, subtler one is the weakness of the assumed identification of subspaces of $\mathcal{H}$ with experimental propositions.

Operational definition of logical connectives

In QM, certain quantities cannot be measured simultaneously. In the set $\mathcal{C}(\mathcal{H})$, they are represented by incompatible questions (associated with non-commutative projectors). Thus, when p and q are two such incompatible experimental questions, neither the conjunction $p \wedge q$ nor the disjunction $p \vee q$ can represent an experimental proposition, for there is no experimental procedure to actually test them. This fact did not escape von Neumann and Birkhoff. On the contrary, one of the two 'suggested questions' put forward by these authors in the conclusion of [32] reads: "What experimental meaning can one attach to the meet and join of two given experimental propositions?".

Nevertheless, the formal structure dug out by von Neumann and Birkhoff is a lattice, where conjunction and disjunction of questions are always defined. Instead of renouncing the applicability of the logical connectives for incompatible questions, operational quantum logic has sought for an appropriate operational definition of these connectives. Various proposals have been made (like Jauch's infinite filters, Jauch and Piron's product question...), but none of them has been consensually accepted so far.

We take this ambiguity as a hint that operational quantum logic is somewhat ill-founded. Its only because quantum logic tries to merge operationally incompatible questions in a single framework that non-classical features such as non-distributivity emerge. This merging, in turn, follows from the implicit requirement of unicity of the logic. But this unicity does not do justice to the notion of complementarity: some quantities are incompatible and cannot be measured simultaneously by the *same* observer, so there is no point to define such things as conjunction and disjunction in this case. From this viewpoint, non-distributivity has little physical meaning.

[1]For the informed reader, the drawbacks we have identified in Rovelli's sketch are the following: (i) Rovelli's reconstruction is limited to finite-dimensional Hilbert spaces (ii) A clear operational definition of logical connectives is absent (iii) The formalism is based on the set of questions pertaining to one system, irrespective of the observers. It is unclear how von Neumann's consistency, which involves several observers, can be implemented in such a scheme.

However, it is a fact that the algebraic structure of $\mathcal{C}(\mathcal{H})$ is at the core of QM. These mathematics are unquestionable. Consequently, what needs to be revised in operational quantum logic is only the physical semantics. Heelan's observation, to which we shall now turn, is crucial in this respect.

Subspaces as questions: a misleading assumption

In a little-known paper of 1970, Heelan argues that quantum logic, understood as the logic of experimental propositions, is misleaded [48]. His argument questions the legitimacy of the initial identification of (closed) subspaces of $\mathcal{H}$ with experimental propositions. Here goes his critique.

von Neumann and Birkhoff's main observation is that experimental propositions of the form '$A \in \Delta$' are represented quantum-mechanically by (closed) subspaces of $\mathcal{H}$. From this fact, they infer that (closed) subspaces are faithful representatives of experimental propositions in the formalism, and therefore encapsulate 'the logic of QM'. But this reasoning is mistaken, because the correspondence between experimental propositions and (closed) subspaces is not 1-to-1. Given a subspace S, there is no univocal experimental proposition associated to it. To understand this point, we need to recall the fundamental teaching of Bohr: *a quantum-mechanical phenomenon is codefined by its physical context.* Thus an experimental proposition '$A \in \Delta$' should actually be understood as 'Given the physical context C, $A \in \Delta$'. Now, it is clear that the specification of an element S of $\mathcal{C}(\mathcal{H})$ does not uniquely define such a complete experimental situation. This would be the case only with the datum of a particular coordinatization of S (or, equivalently, of a complete set of commuting observables for S).

The obfuscation of the physical context in the standard quantum-mechanical approach therefore entails a mistake concerning the locus of the non-classical logic of subspaces of the Hilbert space. This locus *is not* the experimental *events*. Then, what is it?

8.2 A relational meta-language

We have proposed that QM is a theory of relational determinations, in which a physical statement concerning system S is always understood relative to some observer O. Such a statement takes the form 'relative to O, the value of the observable A lies in Δ'. Now, it is clear that *the propositional calculus of such relational statements is classical.* Indeed, *relative to* O, the set of questions of S is modeled by a Boolean algebra. But remember that, in the RQM picture, the theoretical predicates concern the relative descriptions themselves, and not directly the system itself. Accordingly, there should exist, in the quantum formalism, a higher-level 'propositional calculus', made of sentences referring to these relative descriptions. In this *relational meta-language*, different observers' viewpoints are compared and confronted. A proposition stating the consistency of two distinct observers' account of a given sequence of events (von Neumann's consistency) should thus be formulated in this meta-language.

The acknowledgment of these two distinct levels of experimental languages is, we believe, an important ingredient of a future reconstruction of QM. Indeed, the unfamiliar non-distributivity of the Hilbert lattice must be related to the 2nd-order relational language, and not to the relative questions '$A \in \Delta_O$' themselves. In this perspective, we propose that the fundamental difference between classical and quantum mechanics lies in this stratified aspect of the logic ('vertical' structure), rather than in the relations between experimental propositions ('horizontal' structure), as is often claimed.

How do these remarks help us in the reconstruction program advocated by Rovelli? During this internship, I have spent some time on this question, trying to identify a natural framework for these ideas. Unfortunately, the preliminary necessity to digest unfamiliar mathematics, like category theory, made this goal unreachable for me.

However, one thing has appeared clearly: a relational quantum logic should be modeled by a structured set of Boolean algebras. Each Boolean algebra would represent an experimental language, defined relative to some observer, and the relations between the algebras would encapsulate the articulation between different observers' viewpoint. (The latter relations are the aforementioned locus of the non-classicality of quantum logic.) In this direction, I found in Domotor's theory of Boolean atlases [56] an interesting candidate for the sought-for relational quantum logic. The confirmation of this intuition may be the subject of another six-months internship!

Conclusion

The relational interpretation of QM is young, and many important aspects remain to be clarified. Notably, its reconstruction facet is still quite inconclusive, even though RQM appears to have the potential to shed new light on the whole quantum-logical program. In this perspective, we have convinced ourselves that new mathematical tools, like topos theory, may provide the missing formal language in which relational postulates could be naturally formulated. The application of topos theory to QM has been advocated by authors like Isham, Baez and Coecke for a few years. Its relevance to RQM – an admittedly vague claim of ours – deserves to be explored in future works.

On the philosophical side, RQM clearly has radical implications. Among them is the explicit rejection of the natural ontology of 'objects'. Once this move is taken, the need for an alternative answer to the question "what is real?" becomes pressing. The identification of the *relations* between physical systems as the locus of physical determinations is only the first step towards a new definition of reality. But this new reality is nothing less than the horizon of the 'unfinished quantum revolution'.

> "And so on: the sequence of physical theories is open, and the quest for new forms of relativization has largely started. The only precaution to take, in order to leave the track open for the search for future cognitive relativizations, is not to elevate present absolutes to idols (in Bacon's sense), i.e. to avoid taking for an existing thing what really is a temporary functional point of support in the dialectic of intentional aiming and reflexive return which characterizes knowledge." (Bitbol, [10])

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